MATHEMATICS 7

Integers

Distance Learning

Alberta
EDUCATION

Module 5

Mathematics 7

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Mathematics 7
Student Module Booklet
Module 5
Integers
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This document is intended for	
Students	✓
Teachers (Mathematics 7)	✓
Administrators	
Parents	
General Public	
Other	

The Alberta Distance Learning Centre has an Internet site that you may find useful. The address is http://ednet.edc.gov.ab.ca/level3/adlc/300/adlchome.html.

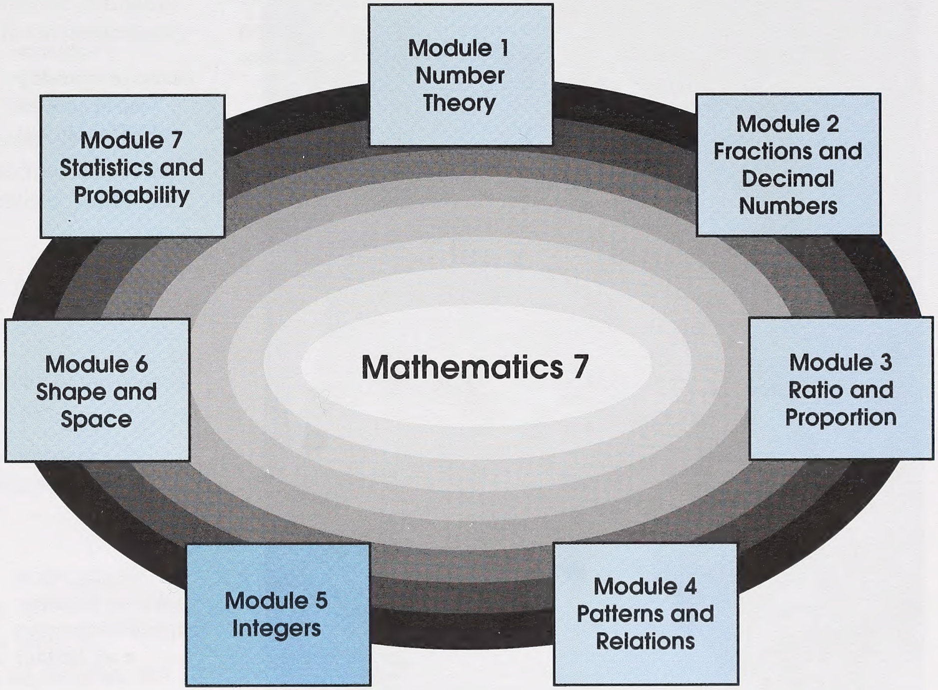
The use of the Internet is optional. Exploring the electronic information superhighway can be educational and entertaining. However, be aware that these computer networks are not censored. Students may unintentionally or purposely find articles on the Internet that may be offensive or inappropriate. As well, the sources of information are not always cited and the content may not be accurate. Therefore, students may wish to confirm facts with a second source.

Welcome

WESTFILE INC.

Welcome to Module 5. We hope you'll enjoy your study of Integers.

Mathematics 7 contains seven modules. Work through the modules in the order given, since several concepts build on each other as you progress in the course.



The document you are presently reading is called a Student Module Booklet. You may find visual cues or icons throughout it. Read the following explanations to discover what each icon prompts you to do.

- Prepare for a problem that will provide a change of topic.

- Prepare for a challenging problem related to the topic of the activity.

- Use the Internet to explore a topic.

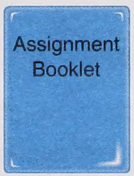
- Use computer software.

- Use a scientific calculator.

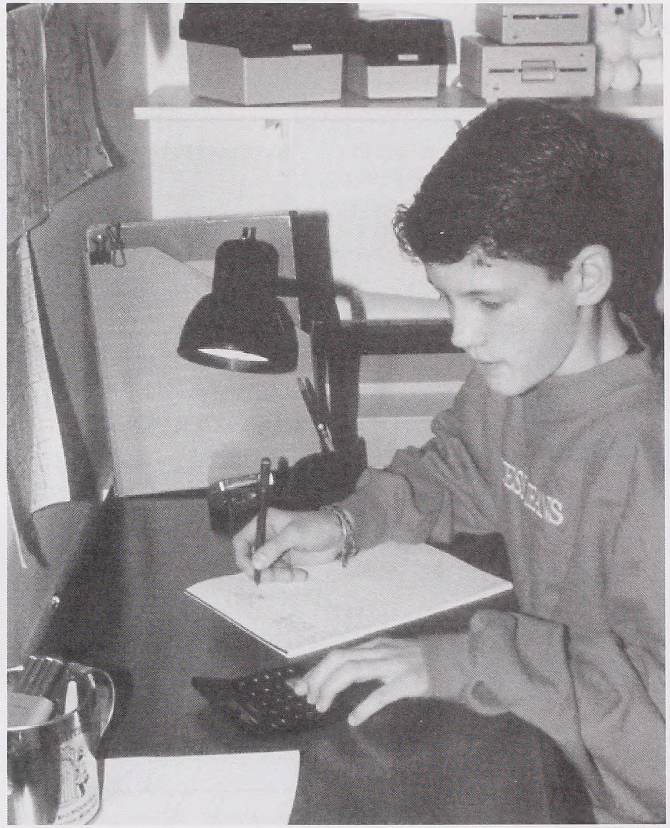
- View a videocassette.

- Pay close attention to important words or ideas.

- Use the suggested answers in the Appendix to correct activities.



- Answer the questions in the Assignment Booklet.



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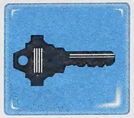
There are no response spaces provided in this Student Module Booklet. This means that you will need to use your own paper for your responses. You should keep your response pages in a binder so that you can refer to them when you are reviewing or studying.

Problem-Solving Skills

One of the exciting features of this course is that you will develop and improve your ability in problem solving. You will need these problem-solving skills many times in your lifetime. Since this course focuses on problem solving, it is important that you understand what a **problem** is.



A problem is a task for which the method of finding the answer (as well as the answer) is not immediately known.

Like any skill, the skill of problem solving must be developed. Problems may or may not involve computation (adding, subtracting, multiplying, and dividing). Some problems are realistic; others are puzzles.

You will have the opportunity in most activities to try a problem-solving challenge.

The  icon is a cue that the problem will be related to the topic of the activity.

The  icon is a cue that the problem will provide a change of topic.

The Four-stage Process

There are four stages that can be used to solve any problem: understanding the problem, developing a plan, trying the plan, and looking back.

Understanding the Problem

In this stage you should expect to feel puzzled. There are various reasons for feeling this way.

- You may not know the meanings of all the words.
- You may not understand the situation in the problem.
- You may be confused by unnecessary information.

Once you understand the problem, you should think about the problem and make an estimate of what the answer should be. This will help you arrive at a reasonable answer.

Developing a Plan

This is where you should decide on the plan of action that you are going to take to solve the problem.

You may consider the following strategies:

- changing your point of view
- using objects
- using diagrams
- making an organized list
- using Venn diagrams
- making a table
- guessing, checking, and revising
- acting out a problem
- working backwards
- simplifying a problem
- finding and applying a pattern
- using elimination
- using truth tables
- using an equation

Note: The Appendix in Module 1 explains these strategies in detail. When you see a problem-solving icon in any module, you should turn to the Appendix in Module 1 and review the problem-solving strategies.

Trying the Plan

In this stage you should try the plan and see if it works.

Be sure to work carefully and record your progress. You are encouraged to use a calculator to help with your calculations.

Note: While trying the plan, you should monitor your progress in order to determine if your plan will lead to a solution. You may find that the plan will not produce a solution, in which case a new plan will have to be developed.

Looking Back

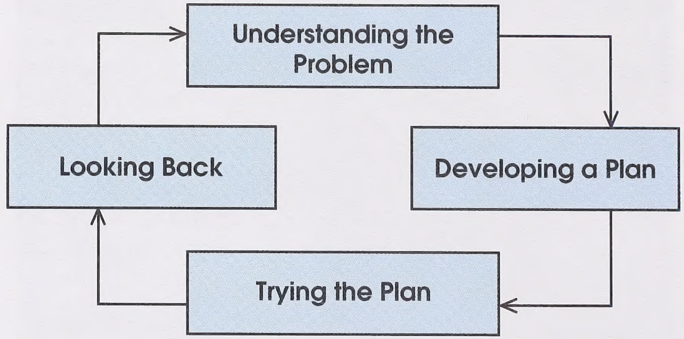
In this stage you should look back at the problem and compare your answer to the estimate you made in the first stage. Restate the problem using your answer.

Ask yourself these questions: "Did my plan work? Is my answer reasonable?"

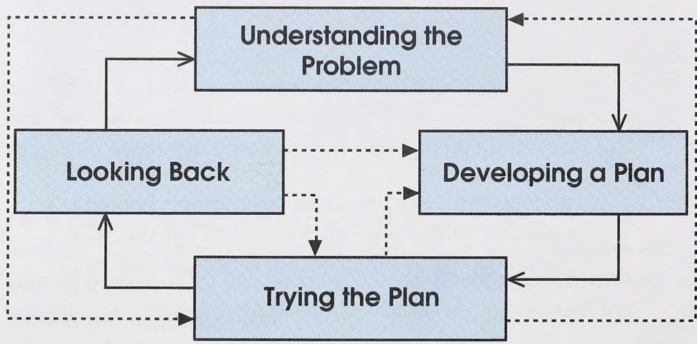
If you did not arrive at an answer, another strategy may work better. If your answer is unreasonable, you may have made errors while trying your plan.

Sequence of Stages

You usually approach a problem in the order outlined in the following diagram.



If you encounter difficulties in your original plan, or if you realize that another strategy will have better results, you may need to return to an earlier stage or use the stages in a different sequence.



CONTENTS

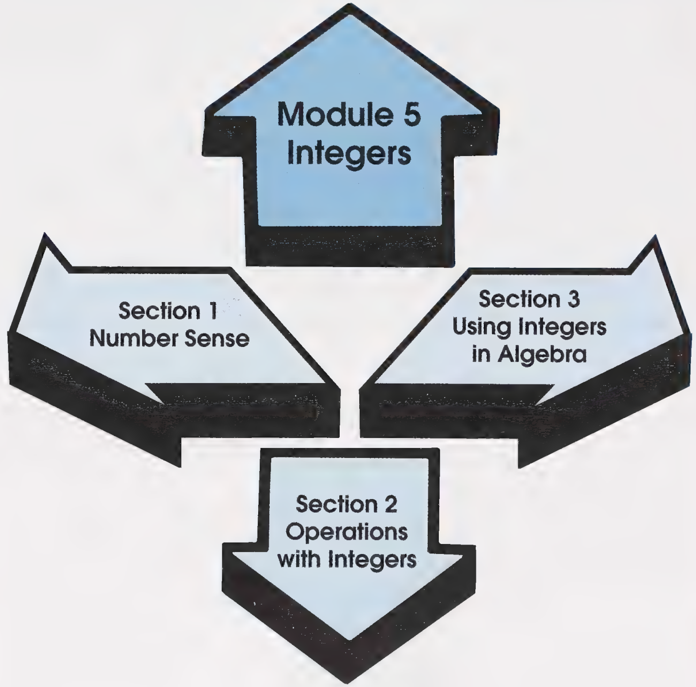
Module Overview

Have you played the board games *Sorry!*® and *Snakes and Ladders*®? In these games you move through a series of spaces to arrive at your goal. Sometimes you suffer a setback and must go backwards. You can think of moving forward three spaces as $+3$, and going backwards three spaces as -3. The numbers $+3$ and -3 are examples of integers.

Integers can be used to describe many other situations which involve direction. Here are a few examples:

- degrees below freezing; degrees above freezing
- parking levels below ground; parking levels above ground
- depth below sea level; height above sea level
- golf scores under par; golf scores above par
- time before an event; time after an event
- a loss; a profit

In this module you will develop a sense for integers. You will compare and order integers, perform operations on integers, and use integers to evaluate algebraic expressions and solve equations. You will continue solving problems and looking for patterns and relationships in numbers.

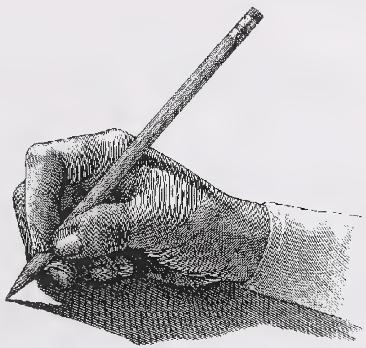


Evaluation

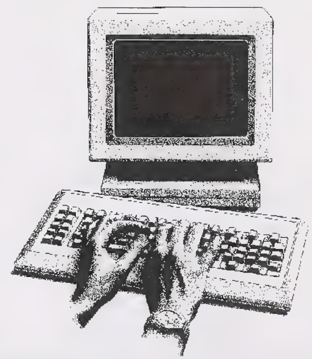
Your mark for this module will be determined by how well you complete the assignments at the end of each section and at the end of the module. In this module you must complete four assignments. The mark distribution is as follows:

Section 1 Assignment	**25 marks**
Section 2 Assignment	**35 marks**
Section 3 Assignment	**20 marks**
Final Module Assignment	**20 marks**
TOTAL	**100 marks**

When doing the assignments, work slowly and carefully. You must do each assignment independently, but if you are having difficulties, you may review the appropriate section in this module booklet.



If you are working on a CML terminal, you will have a module test as well as a module assignment.



Note

There is a final supervised test at the end of this course. Your mark for the course will be determined by how well you do on the module assignments and the supervised final test.

Section 1: Number Sense

Before the early 1800s little was known about the continent of Antarctica. The first expedition to explore this vast land of ice was in 1820. Since then, explorers have discovered mountain ranges that reach almost 5000 m above sea level, and they have charted crevasses that plunge over 2000 m below sea level. Meteorologists have recorded some of the coldest temperatures in the world in Antarctica; the Plateau Station has an average temperature of 57°C below zero.

To appreciate the size of the preceding measurements you must have a sense of these numbers.

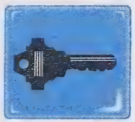
In this section you will represent numbers such as distances above and below sea level, temperatures above and below the freezing point of water, and years B.C. and A.D. as integers. You will compare and order integers. Finally, you will use models to help you visualize integers.

Activity 1: Numbers with Signs

Do you like being outdoors in cold weather, or would you rather be curled up in front of a fire when the temperature is $-20°C$?

You have been using temperatures for many years. You know that room temperature is $+20°C$, the freezing point of water is $0°C$, and $-20°C$ is considered to be a cold day.

Did you know that $+20$, 0, and -20 are **integers**?



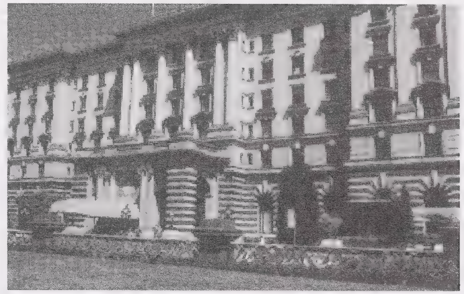
The set of integers includes these numbers: $\{\ldots, -3, -2, -1, 0, +1, +2, +3, \ldots\}$.

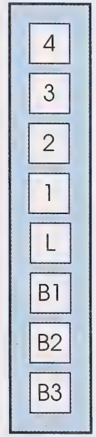
Zero acts like a reference point or a starting point in the set of integers.

- The positive integers are greater than zero. To indicate this, a plus sign (+) is **sometimes** written before the numeral.
- The negative integers are less than zero. A negative integer is **always** written with a minus sign (–) before the numeral.

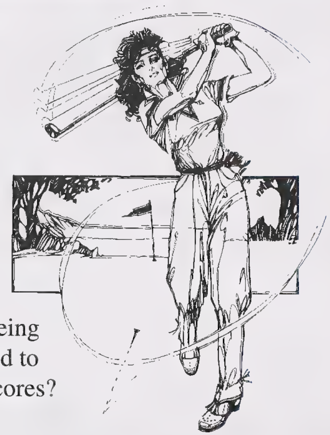
In questions 1 to 7, you will discover that many situations can be represented by integers.

1. In some hotels, parking levels are below ground, the lobby is on the ground level, and the guest rooms are on the floors above the lobby. What integer could be used to indicate each level on this hotel elevator panel?





2. Golf is an outdoor game played with a small hard ball and a set of clubs. The player tries to get the ball into each of the holes on the golf course in as few strokes as possible.

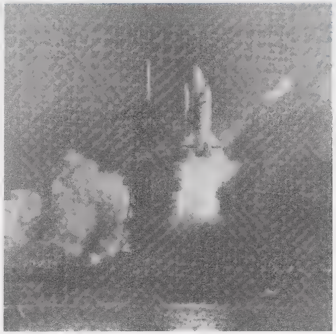


The standard number of strokes set for each hole is called "par." Scores can be expressed as integers with par being zero. What integer could be used to indicate each of the following scores?

a. two under par
b. five over par

3. Time before an event or after an event is sometimes measured. What integer could be used to indicate each of the following times with respect to the launching of a rocket?

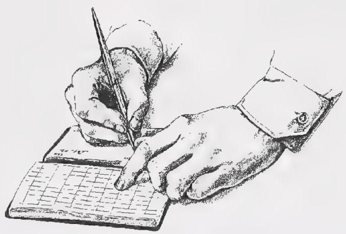
   a. 8 s before blastoff
   b. blastoff
   c. 5 s after blastoff



NASA

4. Accountants use the phrases "in the black" or "in the red" to describe a company's financial position. "In the red" means the company is in debt; it is losing money. "In the black" means the company is making a profit. What integer could be used to indicate the financial position described in each of the following situations?

   a. a profit of $5000
   b. a loss of $800
   c. a surplus of $20
   d. a withdrawal of $75
   e. no change in the balance
   f. a deposit of $100
   g. a deficiency of $45



5. The following chart lists the high and low temperatures for several cities in Canada on a particular day in March.

City	High (°C)	Low (°C)
Calgary	13	−2
Charlottetown	−7	−15
Edmonton	8	−5
Fredericton	−5	−14
Halifax	−6	−13
Inuvik	−26	−36
Ottawa	−4	−6
Prince George	9	0
Saskatoon	6	−4
St. John's	−3	−11
Vancouver	11	5
Whitehorse	−11	−18
Winnipeg	0	−5
Yellowknife	−11	−14

   a. What is the lowest recorded temperature on the chart? For which city?

   b. What is the highest recorded temperature on the chart? For which city?

   c. Which city was warmer on that day, Charlottetown or Halifax?

   d. Which city was colder on that day, Charlottetown or Fredericton?

Check your answers by turning to the Appendix.

The money to run many large companies is obtained from investors who buy stocks or shares in the company. These shares are sold through stock exchanges. If a company makes money, or if the investors expect it to make money, the price of the stock usually rises. If a company loses money, or if the investors are afraid it will lose money, the price of the stock usually falls. The daily happenings of the stock market are published in newspapers.

The following chart shows what happened to various stocks for a particular week in March 1995.

Stock	Volume	High	Low	Close	Change
Willow	717	85	70	85	+5
Windsor Crt	135	20	18	18	−1
Winspear	795	110	100	103	−15
World orgncs	8	17	17	17	−8
Wld Wide O&G	10800	25	12	12	−10
Wstn Keltic	75	51	51	51	
Wstn Logic	10	10	10	10	+1
Wstn Premium	10	18	18	18	+4
Yanks Peak	2310	17	14	14	−4
Yellowjack	718	11	9	10	+1
Young-Shann	10	35	35	35	−5
Yukon Rev	5	14	14	14	
Yuma Gld	1165	77	60	65	−9
Zappa	1654	165	148	160	+15
Zeus Egy Cp	471	97	93	93	−5
Zorah Media	2911	44	38	44	

The chart gives this information.

- Stock: the name of each stock (The names are often abbreviated.)
- Volume: the number of shares (in hundreds of shares) traded that week
- High: the highest price of a share that week (The price is in cents unless a dollar value is given.)
- Low: the lowest price of a share that week
- Close: the price of stock when the stock exchange closed for the week
- Change: the change in the closing price from last week's close to this week's close

Use the preceding chart to answer the following questions.

6. **a.** Which stocks on the chart went down in value from the previous week?

   b. Which stocks on the chart showed no change from the previous week?

   c. Which stock on the chart showed the greatest increase? How much was the increase?

   d. Which stock on the chart showed the greatest decrease? How much was the decrease?

   e. Which stock on the chart showed the largest volume of sales? How many stocks were traded?

   f. Which stock on the chart was selling for the least amount at closing? What was the price?

   g. Which stock on the chart was selling for the greatest amount at closing? What was the price?

Check your answers by turning to the Appendix.

It is often important to know the height, or elevation, of land. In order to compare the heights of different places on Earth, sea level is often used as a starting point. (The levels of each ocean in the world are the same.) For example, the base of Mauna Kea (White Mountain) in the Hawaiian Trough is 5445 m below sea level. The peak of Mauna Kea on Hawaii (Big Island) is 4205 m above sea level.

The following chart lists the highest point and the lowest point on each continent.

Continent	Highest Point (m)	Lowest Point (m)
Africa	Kilimanjaro 5895	Lake Assal −156
Antarctica	Vinson Massif 4897	*unnamed* −2538
Asia	Everest 8848	Dead Sea −400
Australia	Kosclusko 2228	Lake Eyre −16
Europe	El'brus 5642	Casplan Sea −28
N. America	McKinley (Denali) 6194	Death Valley −86
S. America	Aconcagua 6960	Valdés Peninsula −40

The following chart lists the deepest point in each ocean.

Ocean	Lowest Point (m)
Arctic	Eurasia Basin −5122
Atlantic	Puerto Rico Trench −8605
Indian	Java Trench −7258
Pacific	Mariana Trench −10 924

7. Use the given charts to answer the following questions.

   a. What is the lowest point on all the continents?
   b. What is the lowest point in all the oceans?
   c. What is the lowest point on Earth?
   d. What is the highest point on Earth?

Check your answers by turning to the Appendix.

Now Try This

You may be interested to discover that Canada sponsors an observatory on the summit of Mauna Kea. Use the Internet to discover more about this observatory. **Hint:** This is the uniform resource locator (URL) for the Mauna Kea Observatories.

http://www.ifa.hawaii.edu/mko/mko.html

Absolute Value

Every positive integer and negative integer has a sign and an **absolute value**.

The absolute value of an integer is the value of the integer without regard to the sign. The symbol for absolute value is two vertical line segments. Sometimes the absolute value of an integer is called its **magnitude**.

Example

A cold winter day is $-20\,°C$. Room temperature is $+20\,°C$. What is the absolute value of -20 and $+20$?

Solution

$|-20| = 20$

$|-20|$ is read as "the absolute value of -20."

$|+20| = 20$

The absolute value of -20 is 20. The absolute value of $+20$ is 20.

Note: Both $+20\,°C$ and $-20\,°C$ are 20 degrees from the freezing point. However, $+20\,°C$ is 20 degrees above the freezing point and $-20\,°C$ is 20 degrees below the freezing point.

The integers $+20$ and -20 are **opposite integers**.

Two integers are opposite integers if they have the same absolute value but different signs. Sometimes the opposite integer is called the **additive inverse**.

8. Give the absolute value of each of the following integers.

a. -3 **b.** $+7$ **c.** $+5$ **d.** -4

9. Give the opposite integer of each of the following.

a. -2 **b.** $+8$ **c.** $+6$ **d.** -3

Check your answers by turning to the Appendix.

Now Try This

Use a problem-solving strategy to answer the following question.

10. Find a number which has the following characteristics:

- It is less than $+4$.
- It is greater than -2.
- It is even.

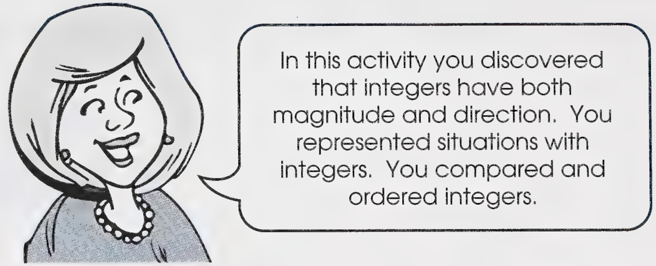
Check your answer by turning to the Appendix.

Did You Know?

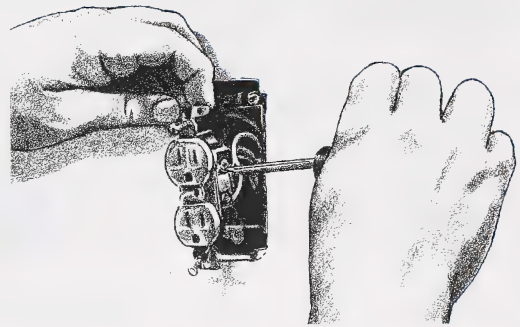
The word **negative** comes from a Latin word meaning “to deny” or “to say no.”

Negative numbers were first used around A.D. 700 by Hindu merchants to indicate a debt or deficiency. Both Hindus and Arabs used negative numbers for many centuries, while early European mathematicians were reluctant to use them.

Although negative numbers were used in a fifteenth-century German publication, many European mathematicians continued to be sceptical. Michael Stifel (1486–1567) called negative numbers “absurd” and René Descartes (1596–1650) called them “false numbers.”



Activity 2: Modelling Integers



Why are electrical wires different colours? Why does a battery have its terminals labelled positive (+) and negative (–)?

There are two kinds of electrical charges—positive charges and negative charges. If a positive charge comes together with a negative charge, the result is no charge or a charge of 0.

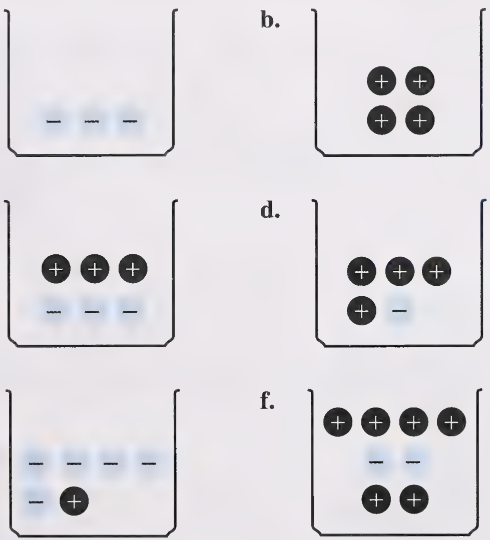
You can use an electrical model to help you understand integers.

View the introduction and the first segment of the video entitled "Positives, Negatives, and Zero" of the program *Integers* from the series *Math Moves*.

Positive and negative counters are provided in the Appendix. You may photocopy this page, glue it to heavy paper (card stock), and cut out the counters. You may prefer to use the objects suggested in the video.

1. State the integers modelled in each of the following diagrams.

a. b. c. d. e. f.



2. Using your counters and a container, build three different models of each of the following integers.

a. 0 **b.** +1 **c.** −3

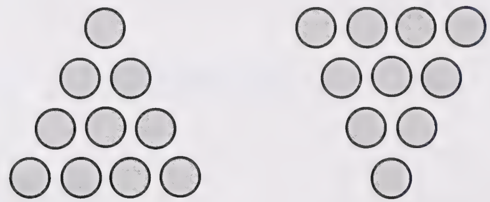
3. Why is it possible to model any integer in any number of ways?

Check your answers by turning to the Appendix.

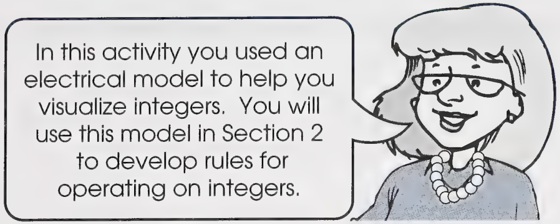
Now Try This

Use a problem-solving strategy to answer the following question.

4. Move three coins in the figure on the left to make it like the figure on the right.



Check your answer by turning to the Appendix.

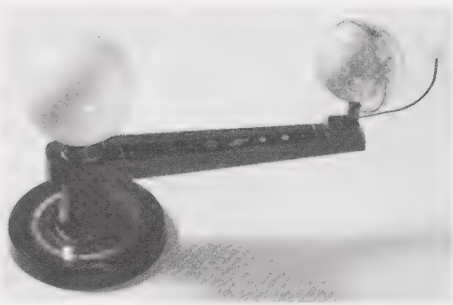


Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

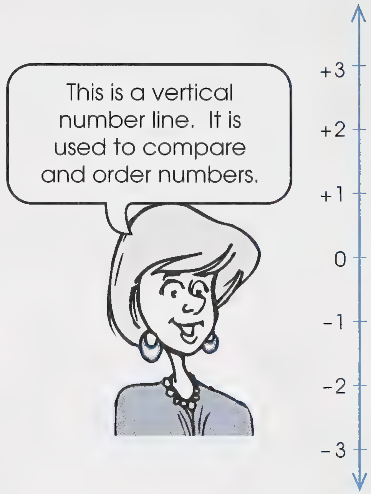
Have you ever used a model like this to help you visualize an eclipse or how day and night are created?



SPECTRUM

Models and diagrams are helpful in visualizing mathematical concepts. In this activity you will use number lines to help you visualize integers.

There are two types of number lines: **vertical number lines** and **horizontal number lines**.

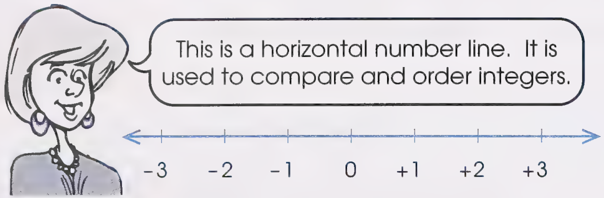


The numbers on a vertical number line increase as you move upward.

- $+3$ is above $+2$, so $+3 > +2$.
- $+1$ is above -2, so $+1 > -2$.
- -1 is above -3, so $-1 > -3$.

The numbers decrease as you move downward.

- $+1$ is below $+3$, so $+1 < +3$.
- -3 is below $+2$, so $-3 < +2$.
- -2 is below -1, so $-2 < -1$.



The numbers on a horizontal line increase as you move from left to right.

- $+4$ is to the right of $+1$, so $+4 > +1$.
- $+1$ is to the right of -2, so $+1 > -2$.
- -1 is to the right of -4, so $-1 > -4$.

The numbers decrease as you move from right to left.

- -4 is to the left of -1, so $-4 < -1$.
- -2 is to the left of $+1$, so $-2 < +1$.
- $+1$ is to the left of $+4$, so $+1 < +4$.

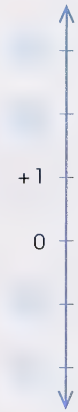
1. Copy and complete each of the following number lines. **Hint:** You can make a number line using a straightedge and compass. Make a line segment with a straightedge. Then open your compass to an appropriate setting and mark off the number of units required.

a.

b.

c.

d.



e.

2. Use < or > to show the relationship between each pair of integers.

a. 6   16   **b.** -5 -3 **c.** -2 -20

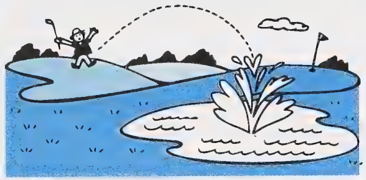
d. -12 1   **e.** $+4$ $+5$ **f.** 0 -3

3. Arrange each group of integers in order from least to greatest.

a. $-4, 8, -3, 12, 0$
b. $+6, 0, -4, -8, +11, +4, -10, -5$
c. $-1, -3, -6, +8, +11, +13, -15$

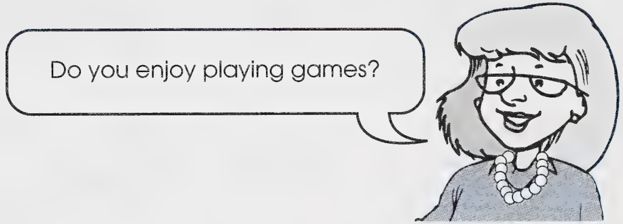
4. The scores of several golfers are shown in the chart. Arrange the scores of the golfers in order from the best score to the worst. (The lowest score is the best score.)

Player	Score
Mario	−2
Soria	+3
Marta	−4
Ian	+5
Petra	−1
Sung	0



5. Cyril watches the weather report one day and finds that the daily high for Winnipeg was $-18°C$ and for Edmonton it was $-12°C$. Because $-18°C$ is colder than $-12°C$, he concludes that the integer -18 is greater than the integer -12. Is Cyril's conclusion true? Explain.

Check your answers by turning to the Appendix.



Games are helpful in developing skills in mathematics.

"Integer War" is a great game for reinforcing the comparison of integers as well as the concept of absolute value.

Find Set 1 and Set 2 of "Integer War" cards in the Appendix. Make photocopies of the pages, glue the photocopies to heavy paper (use two colours to identify the sets), and cut out the cards.

6. Play "Integer War" with a friend. You will each need an identical deck of 30 cards.

How to Play "Integer War"

Each player takes a complete set of 30 cards, puts aside the four "absolute value" cards for the time being, shuffles the remaining cards, and places the shuffled cards face down in a pile on the table.

Each player turns up the top card from his or her deck and compares it with the other player's card. The player with the greater card takes both cards and puts them face down on the table in a second pile.

When there is a tie, both players continue turning over cards until one of them comes out greater. That player then keeps all cards that were turned over during that round.

Four times during the game, a player can use an absolute value card after the two cards in play have been turned over. When this card is played, players compare the absolute value of the integers on the cards.

Play continues in this manner. When all cards from a player's first pile are used, the cards from the second pile are then shuffled and placed back into play.

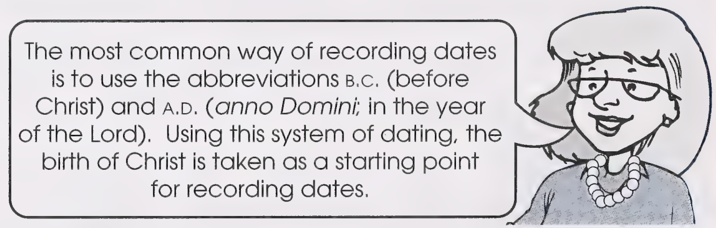
The winner of the game is the person who takes all the cards.

Note: You can vary this game by making new cards with different integers on them.

Enrichment

How old do you think these carvings are? Did you know scientists can use radioactive dating to indicate the age of an artifact?

The abbreviation B.P. (before the present) is used to indicate the age of the artifacts. For example, radioactive datings show that the Norse settled in Newfoundland around 1000 B.P.



Example 1

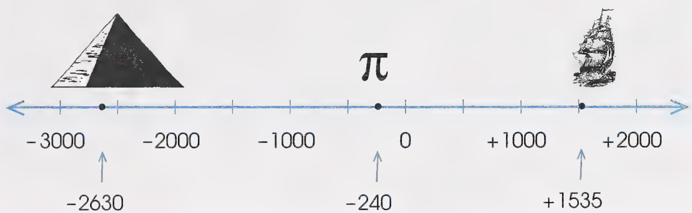
The following historical dates are given using the B.C./A.D. system of recording time.

- In 2630 B.C., Egypt's first pyramid was built.
- In 240 B.C., Archimedes determined that π is between $3\frac{10}{71}$ and $3\frac{10}{70}$.
- In A.D. 1535, Jacques Cartier made a voyage to what is now Canada.

These historical dates can be represented by integers.

- The date 2630 B.C. can be written as the integer -2630.
- The date 240 B.C. can be written as the integer -240.
- The date A.D. 1535 can be written as the integer $+1535$.

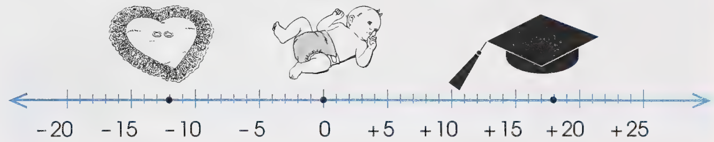
These integers can be shown on a time line.



Note: There is no such date as 0. On the time line, the integer 0 simply shows a starting point.

Example 2

The following time line gives significant events for Justin.



Notice that the date of Justin's birth is represented by 0. The marriage of his parents is represented by -12; it occurred 12 years before his birth. The year that he will graduate is represented by $+18$; 18 years after his birth.

1. Use the time line for Justin in Example 2 to answer this question. **Note:** Justin was born in 1980.

a. Justin's sister Becky was born in 1978. What integer would represent the year of Becky's birth?

b. Justin's brother Clyde was born in 1983. What integer would represent the year of Clyde's birth?

c. Justin began school in 1985. What integer would represent the date of his starting school?

2. Use the year of your birth as 0 and draw a time line to show the following significant events. **Hint:** See the Extra Help to discover how to use a compass to make a number line.

- Calgary Winter Olympics (1988)
- Armstrong and Aldrin—first walk on the Moon (1969)
- start of twenty-first century (2001)

3. A coin expert examines three coins. One is dated 1885. A second is dated 250 B.C. The third is dated -50. The coin expert concludes that the oldest coin is the one dated 1885. Do you agree? Why or why not?

Check your answers by turning to the Appendix.

Did You Know?

Egyptian Problem Solving[1]

The ancient Egyptians had a special way of finding the answers to certain kinds of number riddles. It was a method of guessing the answer and then checking to see how good the guess was.

For example, if 3 times a number is added to the number, the result is 24. What is the number?

An Egyptian of long ago might have guessed 2. However, 3 times 2 added to 2 gives $6+2$, or 8. The answer ought to be 24, which meant that 8 was three times too small. The person would then reason that the guess of 2 was also three times too small. So the number ought to be 3×2, or 6. The person would check to see that 3 times 6 added to 6 gives $18+6$, or 24, as it should.

The method of the Egyptians was used by other ancient peoples. In fact, it was used in the early arithmetic books in North America. At that time, it was called the method of **false position** because the guess put one in a “false position.”

Now Try This

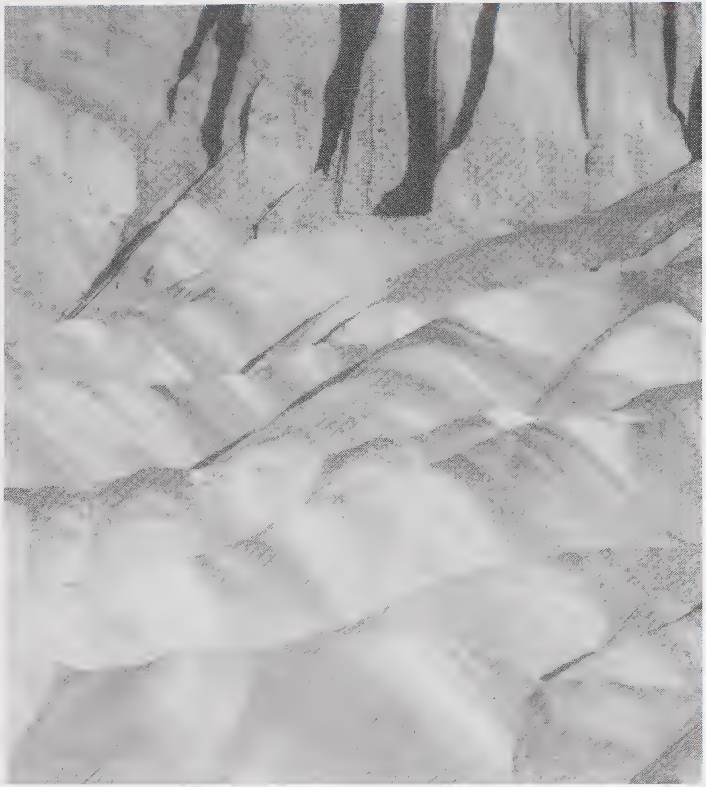
Use the method of false position to answer the following question.

4. The sum of 5 times a number and 2 times the same number is 84. What is the number? **Hint:** Make a guess of 4, and then adjust the guess.

Check your answer by turning to the Appendix.

[1] Reprinted with permission from *Mathematical History: Activities, Puzzles, Stories, and Games*, by Merle Mitchell. Copyright 1978 by the National Council of Teachers of Mathematics.

Conclusion

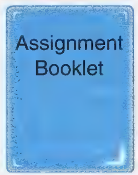


In this section you represented numbers such as the height above or the depth below sea level, temperatures above or below the freezing point of water, and years B.C. or A.D. as integers. You compared and ordered integers, and you used models to help you visualize integers.

Does a sense of integers help you to appreciate the size of measurements such as the ones in the following paragraph?

Death Valley is located in southeastern California. It is the hottest and driest place in North America. The temperature there rarely falls below the freezing point and the record high is 57°C. Death Valley is also the lowest point in North America. Parts of Death Valley are 82 m below sea level. It is estimated that a body of water filled the valley about 50 000 years ago, and that the water evaporated over time.

Assignment



You are now ready to complete the module assignment for Section 1.

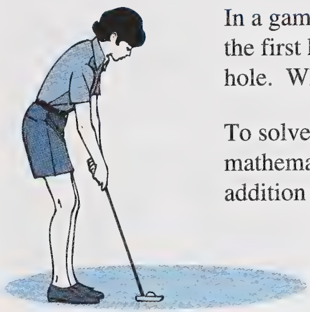
Section 2: Operations with Integers

Hockey is a very popular sport in North America. It is played at all levels of expertise, from the backyard skating rink with neighbourhood friends to professional leagues with players who make hockey their career.

In competitive hockey, statistics are kept on the number of goals players score and the number of assists they make. These records are used by players and coaches to improve the quality of play and the overall success of the team. Because each player does not have the same opportunity to score or assist on goals, a plus-minus system is used. Each player, except the goal tender, is credited with a point for, $+1$, if they are on the ice when their team scores. Players on the ice when the opposition scores a goal are credited with a point against, -1. Each player is rated over a number of games by adding the points for and the points against.

In this section you will use learning aids to visualize the operations of addition, subtraction, multiplication, and division with integers. You will solve problems with integers. You will use the rules for order of operations to evaluate expressions with integers. You will use a calculator, a paper-and-pencil method, and mental computation.

Activity 1: Adding Integers



In a game of golf, Jenny scored 2 over par on the first hole and 1 under par on the second hole. What was her score after two holes?

To solve this problem, you can write a mathematical expression involving the addition of integers.

The expression can be written horizontally or vertically.

$$\underset{\text{first score}}{(+2)}+\underset{\text{second score}}{(-1)}$$

$$\begin{array}{rl} (+2) & \leftarrow \text{first score} \\ \underline{+\ (-1)} & \leftarrow \text{second score} \end{array}$$

Notice that brackets have been placed around the integers in the mathematical expression. This was done to emphasize that the numbers being added are integers.

1. Write a mathematical expression to describe each of the following situations. (Do not calculate any answers at this point.)

a. An airplane descends 400 m and then rises 100 m. How much did the airplane descend or rise altogether?

b. The temperature fell 3°C in one hour and then fell a further 4°C in the second hour. How much did the temperature rise or fall altogether?

c. Frank deposited $420 into his bank account and then withdrew $100. How much did his bank account balance rise or fall altogether?

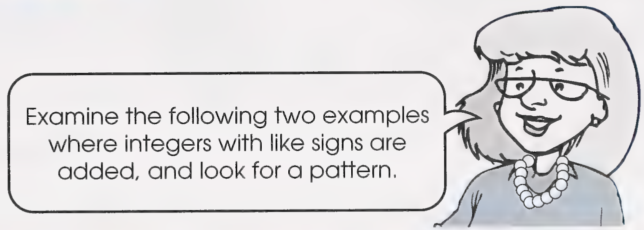
Check your answers by turning to the Appendix.

Note: You will solve the problems in question 1 at the end of this activity.

Adding Integers with Like Signs

To visualize the addition of integers with like signs, gather your counters and a container, view the segment entitled "Adding Integers 1" of the program *Integers* from the series *Math Moves*, and do the video assignment.

Check your answers by viewing the video.

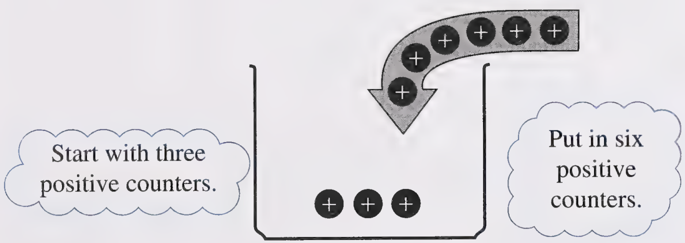


Example 1

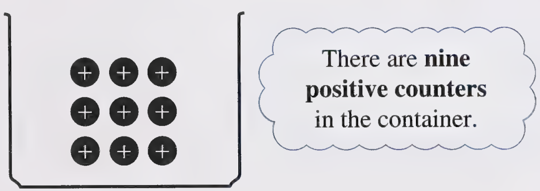
Use modelling to evaluate the expression $(+3)+(+6)$.

Solution

Step 1: Model the expression. Start with three positive counters in the container; then put in six more positive counters.



Step 2: Find the sum. **Hint:** Are the counters in the container negative or positive? How many are there?



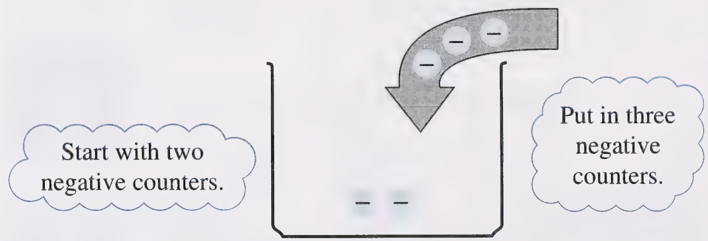
$\therefore\ (+3)+(+6)=+9$

Example 2

Use modelling to evaluate the expression $(-2)+(-3)$.

Solution

Step 1: Model the expression. Start with two negative counters in the container; then add three negative counters.



Step 2: Find the sum. **Hint:** Are the counters in the container negative or positive? How many are there?

$\therefore\ (-2)+(-3)=-5$

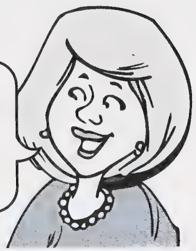
2. Use Example 1 and Example 2 to answer the following questions.

a. What do you notice about the absolute value of the sum of two integers with like signs?

b. What do you notice about the sign of the sum of two integers with like signs?

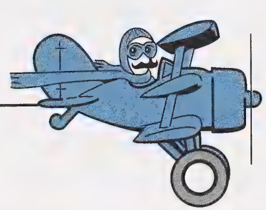
Check your answers by turning to the Appendix.

When you add integers, you must consider the signs of the integers and the absolute values of the integers.



The following rule will help you mentally add integers with like signs.

To find the sum of integers with **like** signs, add the absolute values of the integers and use the sign of the integers.



Example 3

$$\begin{array}{r} (+7) \\ +\ (+1) \\ \hline \end{array}$$

Both integers are positive. Add the absolute values: $7+1=8$. Use the sign of the integers: $+8$.

$\therefore\ (+7)+(+1)=+8$

Example 4

$(-3)+(-6)$

Both integers are negative. Add the absolute values: $3+6=9$. Use the sign of the integers: -9.

$\therefore\ (-3)+(-6)=-9$

3. Mentally compute each of these sums. Write the answers in your notebook.

a. $(+2)+(+5)$

b. $(-3)+(-8)$

c. $\begin{array}{r} (-5) \\ +\ (-4) \\ \hline \end{array}$

d. $\begin{array}{r} (+4) \\ +\ (+9) \\ \hline \end{array}$

4. Franz increased the speed of his car by 30 km/h. Then he accelerated another 45 km/h. How much did he increase or decrease the speed of the car altogether?

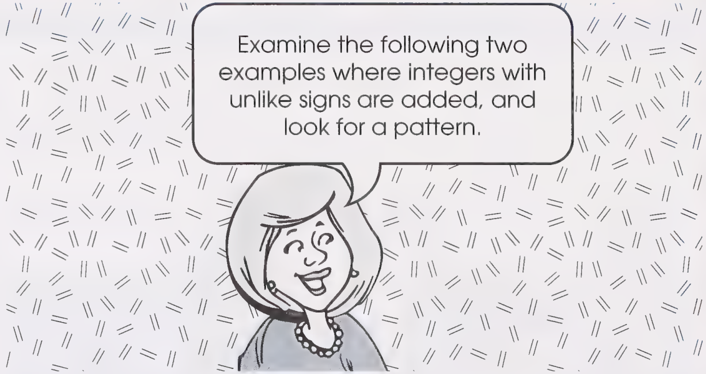
5. Jasmine made a bill payment of $50. Then she withdrew $40 from her bank account. How much did Jasmine's account increase or decrease altogether?

Check your answers by turning to the Appendix.

Adding Integers with Unlike Signs

To visualize the addition of integers with unlike signs, gather your counters and a container, view the segment entitled "Adding Integers 2" of the program *Integers* from the series *Math Moves*, and do the video assignment.

Check your answers by viewing the video.

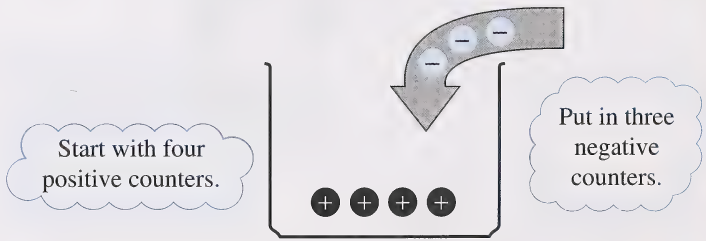


Example 1

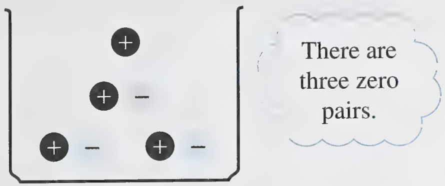
Use modelling to evaluate the expression $(+4)+(-3)$.

Solution

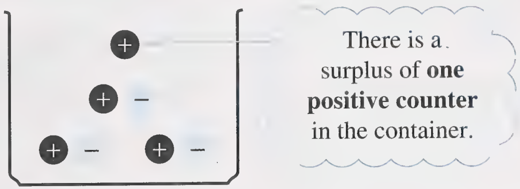
Step 1: Model the expression. Start with four positive integers; then add three negative integers.



Step 2: Because there is a combination of positive and negative counters in the container, rearrange the counters making as many zero pairs as possible.



Step 3: Find the sum. **Hint:** Are the surplus counters in the container positive or negative? How many are there?



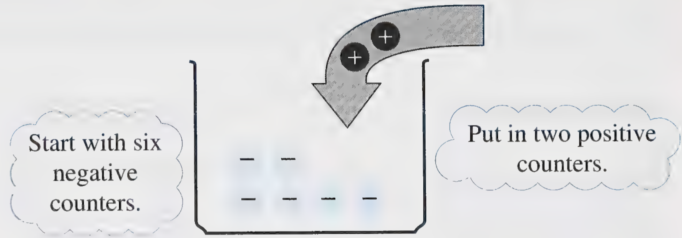
$\therefore\ (+4)+(-3)=+1$

Example 2

Use modelling to evaluate the expression $(-6)+(+2)$.

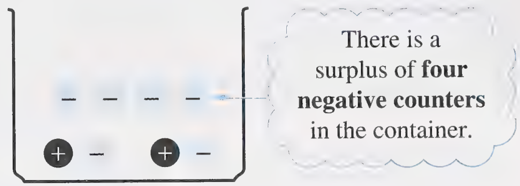
Solution

Step 1: Model the expression. Start with six negative counters; then add two positive counters.



Step 2: Because there is a combination of positive and negative counters in the container, rearrange the counters making as many zero pairs as possible.

Step 3: Find the sum. **Hint:** Are the surplus counters in the container positive or negative? How many are there?



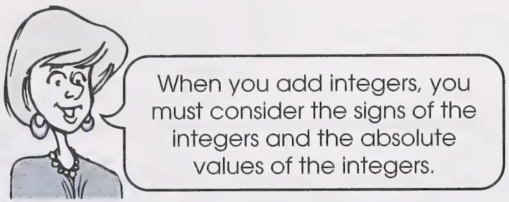
$\therefore\ (-6)+(+2)=-4$

6. Use Example 1 and Example 2 to answer the following questions.

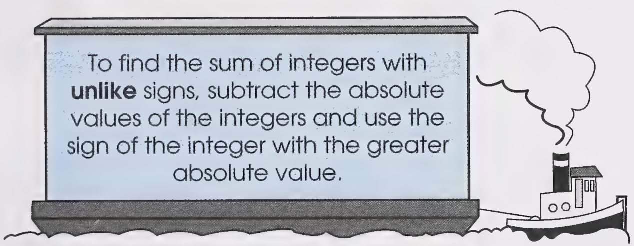
   a. What do you notice about the absolute value of the sum of integers with unlike signs?

   b. What do you notice about the sign of the sum of integers with unlike signs?

Check your answers by turning to the Appendix.

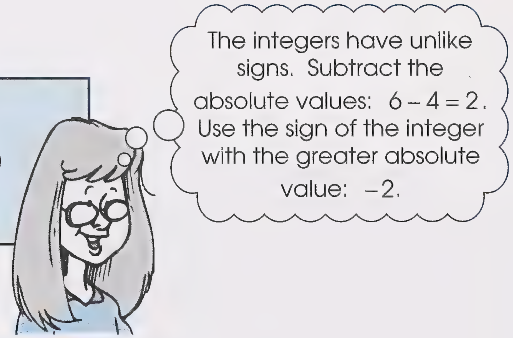


The following rule will help you mentally add integers with unlike signs.



Example 3

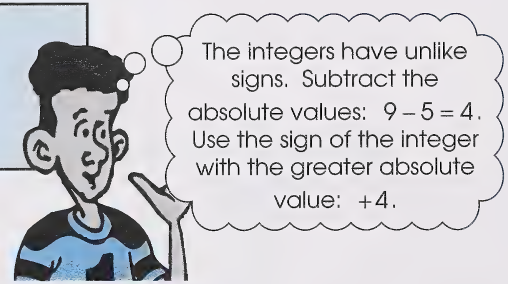
$(-6)+(+4)$



$\therefore\ (-6)+(+4)=-2$

Example 4

$$\begin{array}{r} (+9) \\ +\ (-5) \\ \hline \end{array}$$



$\therefore\ (+9)+(-5)=+4$

7. Mentally compute each of these sums. Write the answers in your notebook.

a. $(-4)+(+7)$ b. $(+2)+(-5)$ c. $(-6)+(+8)$

d. $\begin{array}{r} (+2) \\ +\ (-2) \\ \hline \end{array}$ e. $\begin{array}{r} (-8) \\ +\ (+3) \\ \hline \end{array}$ f. $\begin{array}{r} (-1) \\ +\ (+5) \\ \hline \end{array}$

8. A car was driven forward 45 m and then backward 10 m. How far did the car move forward or backward altogether?

9. The temperature rose 3°C and then fell 8°C. What was the total rise or fall in temperature?

Check your answers by turning to the Appendix.

Putting the Rules Together

- When finding the sum of integers with **like** signs, add the absolute values of the integers and use the sign of the integers.

$$\begin{array}{r} (+7) \\ +\ (+1) \\ \hline +8 \end{array} \qquad (-3)+(-6)=-9$$

- When finding the sum of integers with **unlike** signs, subtract the absolute values of the integers and use the sign of the integer with the greater absolute value.

$$\begin{array}{r} (+9) \\ +\ (-5) \\ \hline +4 \end{array} \qquad (-6)+(+4)=-2$$

10. Solve the problems given in question 1 of this activity.

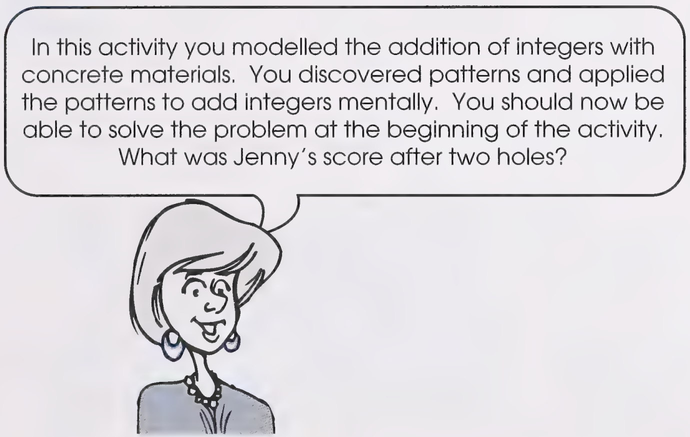
Check your answers by turning to the Appendix.

Now Try This

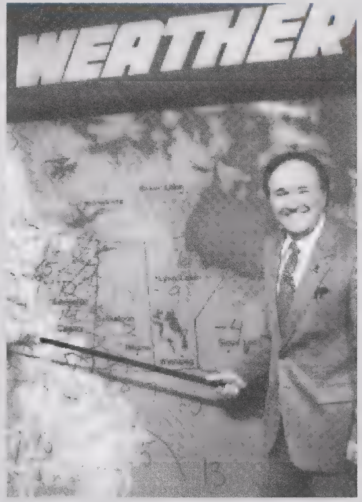
Use a problem-solving strategy to answer the following question.

11. Ramona wrote a five-digit number. The number contains the digits 1, 2, 3, 4, and 5; but the digits are not in this order. The digit 1 is before 3, but after 4. The digit 2 is after 4, but before 1. The digit 5 is after 2, but before 3. If 5 is not the third digit, what number did Ramona write?

Check your answer by turning to the Appendix.



Activity 2: Subtracting Integers



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A chinook is a warm, dry wind that sometimes blows down the eastern side of the Rocky Mountains and across the prairies of western Canada. It can raise the temperature by several degrees in a short time.

For example, because of a chinook, the temperature in Medicine Hat was -8°C at 11:00 and 15°C at 13:00. What was the magnitude and direction of the temperature change?

In order to solve this problem, you can write a mathematical expression involving the subtraction of integers. The expression can be written horizontally or vertically.

$$\underset{\text{temperature at 13:00}}{\underset{\uparrow}{(+15)}} - \underset{\text{temperature at 11:00}}{\underset{\uparrow}{(-8)}}$$

$$\begin{array}{rl} (+15) & \leftarrow \text{temperature at 13:00} \\ \underline{-\quad(-8)} & \leftarrow \text{temperature at 11:00} \end{array}$$

1. Write a mathematical expression to describe each of the following situations. (Do not calculate any answers at this point.)

   a. The opening share price of Allison Products was \$5. The closing share price was \$2. What was the direction and the magnitude of the price change?

   b. A hotel has underground parking, a lobby on the ground floor, and guest rooms on the floors above the lobby. The elevator travelled from the second parking level to the third floor above the lobby. What was the direction and magnitude of the change in position of the elevator?

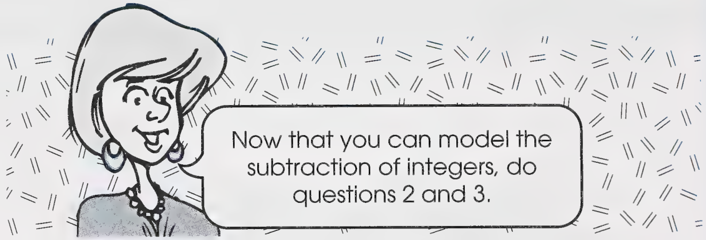
   c. A submarine is 3 km below sea level at 9:00. It is 4 km below sea level at 10:00. What is the direction and magnitude of the change in position of the submarine?

Check your answers by turning to the Appendix.

Note: You will solve the problems in question 1 at the end of this activity.

To visualize the subtraction of integers, gather your counters and a container, view the segments entitled "Subtracting Integers 1" and "Subtracting Integers 2" of the program *Integers* from the series *Math Moves*, and do the video assignments.

Check your answers by viewing the video.



2. **a.** Model the expression $(-5)-(-3)$; then find the difference.

   b. Model the expression $(-5)+(+3)$; then find the sum.

   c. What is the same in the expressions $(-5)-(-3)$ and $(-5)+(+3)$?

   d. What is different in the expressions $(-5)-(-3)$ and $(-5)+(+3)$?

   e. What do you notice about the value of expression $(-5)-(-3)$ and the value of the expression $(-5)+(+3)$?

3. **a.** Model the expression $(+1)-(+2)$; then find the difference.

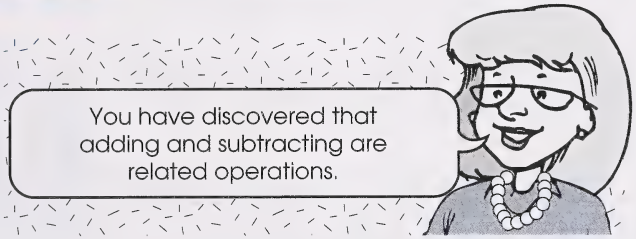
   b. Model the expression $(+1)+(-2)$; then find the sum.

   c. What is the same in the expressions $(+1)-(+2)$ and $(+1)+(-2)$?

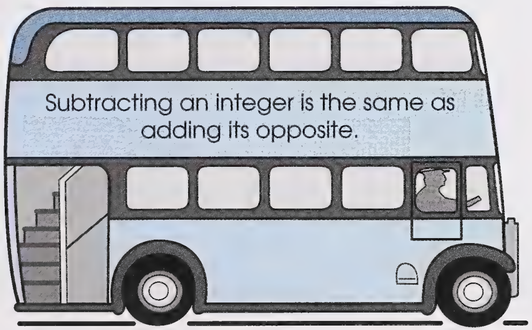
d. What is different in the expressions $(+1)-(+2)$ and $(+1)+(-2)$?

e. What do you notice about the value of the expression $(+1)-(+2)$ and the value of the expression $(+1)+(-2)$?

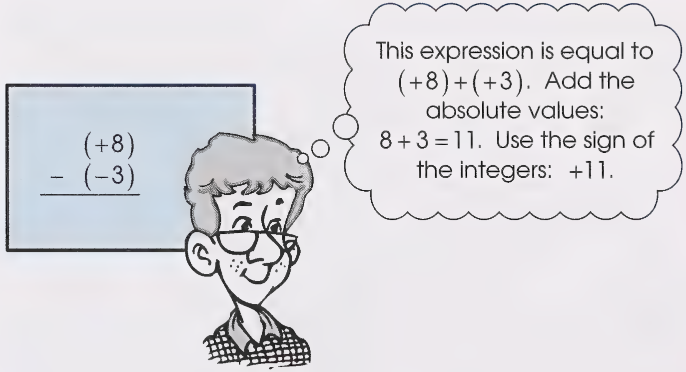
Check your answers by turning to the Appendix.



This rule will help you subtract integers mentally.

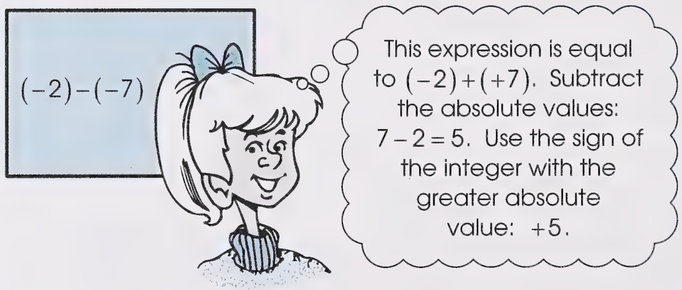


Example 1



$\therefore\ (+8)-(-3)=+11$

Example 2



$\therefore\ (-2)-(-7)=+5$

4. Mentally compute each of the following differences. Write the answers in your notebook.

a. $(-7)-(-9)$ b. $(+4)-(+6)$ c. $(-3)-(+5)$

d. $\begin{array}{r} (+6) \\ -\ (-3) \\ \hline \end{array}$ e. $\begin{array}{r} (-8) \\ -\ (-2) \\ \hline \end{array}$ f. $\begin{array}{r} (+9) \\ -\ (+7) \\ \hline \end{array}$

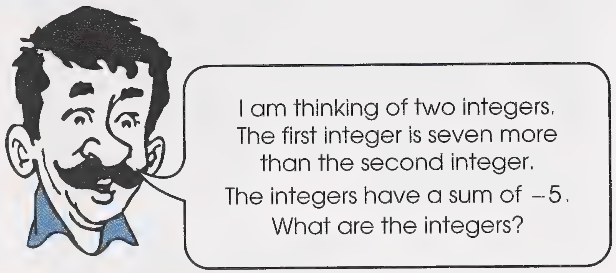
5. Solve the problems in question 1 of this activity.

Check your answers by turning to the Appendix.

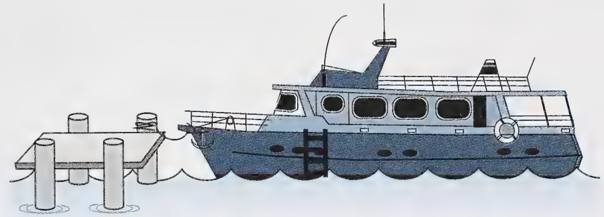
Now Try This

Use a problem-solving strategy to answer the following questions.

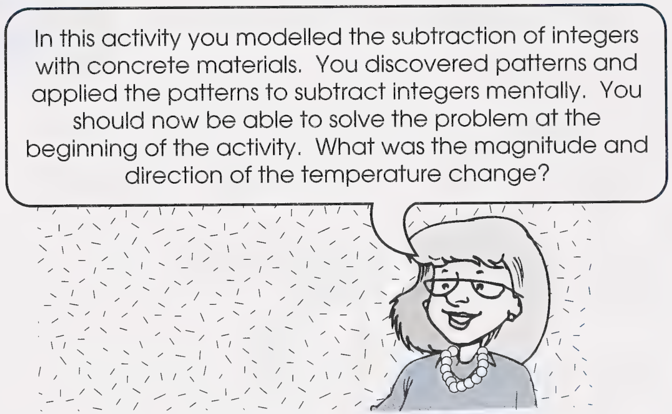
6.



7. A ladder hangs over the side of a ship. The ship is tied to a dock and 5 m of the ladder is submerged. If the tide rises 2 m, how much of the ladder is submerged?



Check your answers by turning to the Appendix.



Activity 3: Multiplying Integers

Michelle likes to climb the cliffs of Nova Scotia's Bay of Fundy. At low tide, the cliffs are exposed, and the water line can be seen on the cliffs.

If Michelle's present position is at the water line, and she is moving up the cliff at a rate of 2 m per minute, what was her position five minutes ago?

To solve this problem you can write a mathematical expression involving the multiplication of integers. The expression can be written horizontally or vertically.

$$(-5)\times(+2)$$

number of minutes in the past

distance in an upward direction in one minute

$$\begin{array}{r} (-5) \\ \times\ (+2) \\ \hline \end{array}$$

number of minutes in the past

distance in an upward direction in one minute

1. Write a mathematical expression to describe each of the following situations about Michelle and her rock climbing. (Do not calculate any answers at this point.)

   a. If Michelle's present position is at the water line, and she is climbing up the cliff at a rate of 2 m per minute, what will her position be in three minutes?

   b. If Michelle's present position is at the water line, and she is climbing down the cliff at a rate of 2 m per minute, what was her position three minutes ago?

   c. If Michelle's present position is at the water line, and she is climbing down the cliff at a rate of 2 m per minute, what will her position be in three minutes?

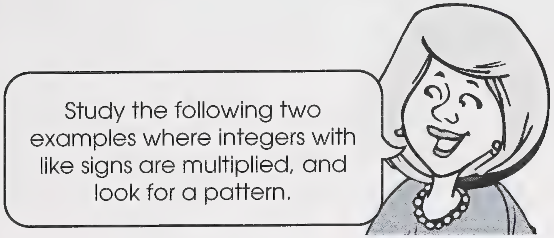
Check your answers by turning to the Appendix.

Note: You will solve the problems in question 1 at the end of this activity.

To visualize the multiplication of integers, gather your counters and a container, view the segment entitled "Multiplying Integers" of the program *Integers* from the series *Math Moves*, and do the video assignments.

Check your answers by viewing the video.

When you model products, remember that the first factor tells you the number of groups to put in or take out of the container. The second factor tells you the kind and number of counters in each group.

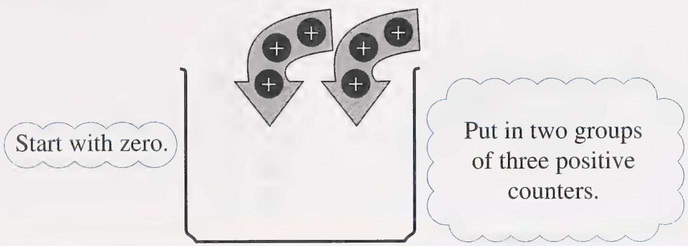


Example 1

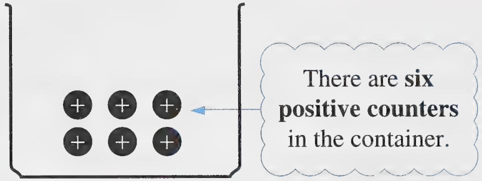
Use modelling to evaluate the expression $(+2)\times(+3)$.

Solution

Step 1: Model the expression. **Hint:** The expression $(+2)\times(+3)$ means, "If two groups of three positive counters are put into the container, what is the result?"



Step 2: Find the product. **Hint:** What kind of counters are in the container? How many are there?



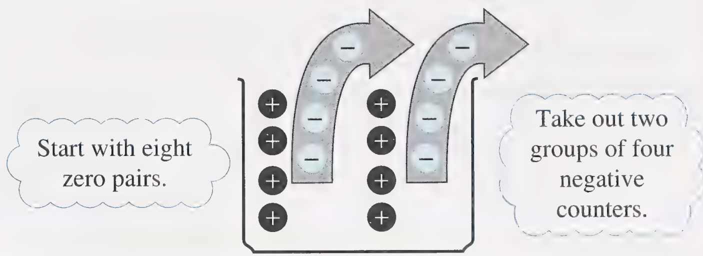
$\therefore\ (+2)\times(+3)=+6$

Example 2

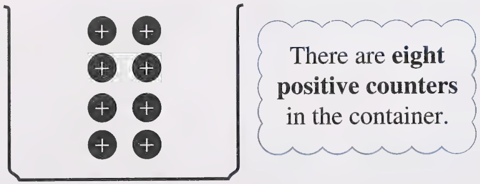
Use modelling to evaluate the expression $(-2)\times(-4)$.

Solution

Step 1: Model the expression. **Hint:** The expression $(-2)\times(-4)$ means, "If two groups of four negative counters are taken out of the container, what is the result?" To take out two groups of four negative counters, you will need eight zero pairs in the container.



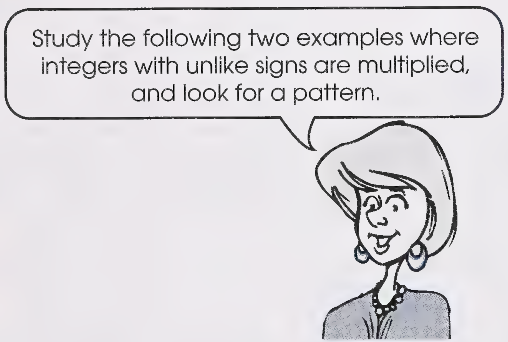
Step 2: Find the product. **Hint:** Are the counters in the container positive or negative? How many are there?



$\therefore\ (-2)\times(-4)=+8$

2. Use Example 1 and Example 2 to answer this question: What do you notice about the sign of the product of two integers with like signs?

Check your answer by turning to the Appendix.

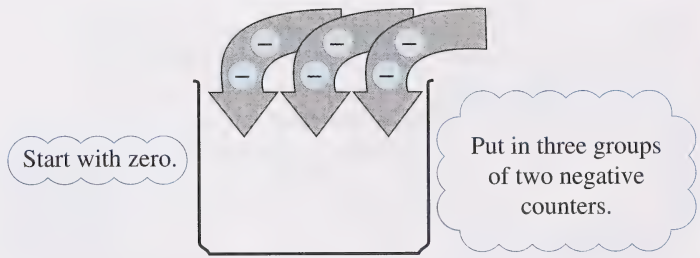


Example 3

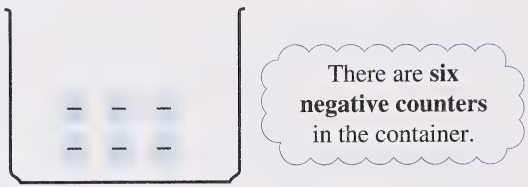
Use modelling to evaluate the expression $(+3)\times(-2)$.

Solution

Step 1: Model the expression. **Hint:** The expression $(+3)\times(-2)$ means, "If three groups of two negative counters are put into the container, what is the result?"



Step 2: Find the product. **Hint:** Are the counters in the container positive or negative? How many are there?



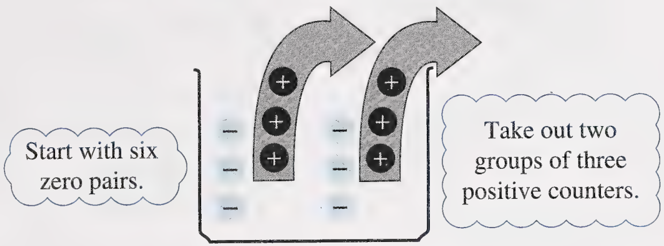
$\therefore\ (+3)\times(-2)=-6$

Example 4

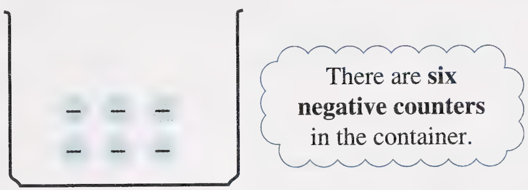
Use modelling to evaluate the expression $(-2)\times(+3)$.

Solution

Step 1: Model the expression. **Hint:** The expression $(-2)\times(+3)$ means, "If two groups of three positive counters are taken out of the container, what is the result?" To take out two groups of three positive counters, you will need six zero pairs in the container.



Step 2: Find the product. **Hint:** Are the counters in the container positive or negative? How many are there?

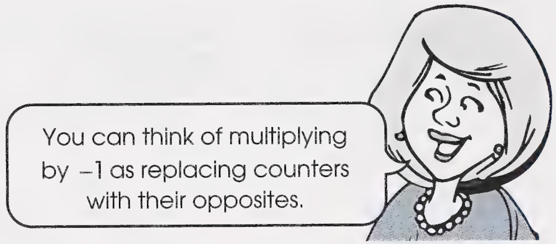


$\therefore\ (-2)\times(+3)=-6$

3. Use Example 3 and Example 4 to answer this question: What do you notice about the sign of the product of two integers with unlike signs?

Check your answer by turning to the Appendix.

If you are still not convinced that the product of two integers with like signs is a positive integer and the product of two integers with unlike signs is a negative integer, you may find examples 5 to 9 helpful.



Example 5

Use modelling to evaluate the expression $(-1)\times(+4)$.

Solution

Step 1: Model $+4$.

Step 2: To model the expression $(-1)\times(+4)$, replace the four positive counters with their opposites.

$-\ -\ -\ -$

There are now **four negative counters**.

$\therefore\ (-1)\times(+4)=-4$

Example 6

Model the expression $(-1)\times(-4)$; then find the product.

Solution

Step 1: Model -4.

$-\ -\ -\ -$

Step 2: To model $(-1)\times(-4)$, replace the four negative counters with their opposites.

$+\ +\ +\ +$

There are now **four positive counters**.

$\therefore\ (-1)\times(-4)=+4$

4. **a.** Model the expression $(-1)\times(+5)$; then find the product.

b. Model the expression $(-1)\times(-6)$; then find the product.

Check your answers by turning to the Appendix.

In Module 2, you discovered that multiplication can be shown as an array. You can also use arrays to model integers.

Example 7

Use modelling to evaluate the expression $(+2)\times(+3)$.

Solution

The expression $(+2)\times(+3)$ can mean two rows of three positive counters.

$+\ +\ +$
$+\ +\ +$

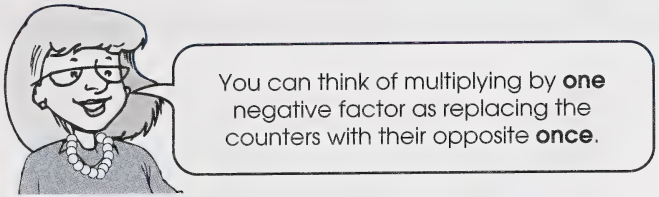
There are **six positive counters**.

$\therefore\ (+2)\times(+3)=+6$

5. **a.** Model the expression $(+3)\times(+4)$; then find the product.

b. Model the expression $(+5)\times(+5)$; then find the product.

Check your answers by turning to the Appendix.

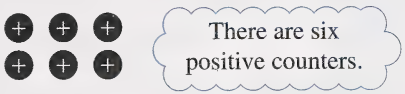


Example 8

Use modelling to evaluate the expression $(-2)\times(+3)$.

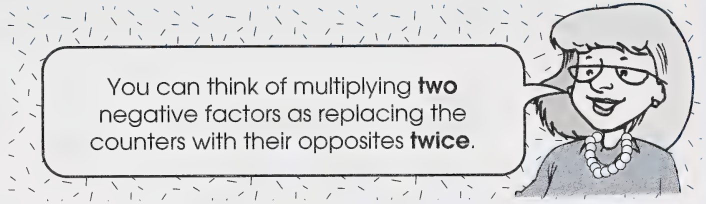
Solution

Step 1: Model $(+2)\times(+3)$.



Step 2: There is **one** negative factor in the expression $(-2)\times(+3)$. So, replace the six positive counters with their opposites **once**.

$\therefore\ (-2)\times(+3)=-6$

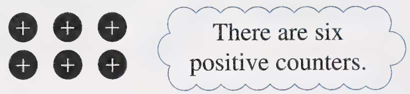


Example 9

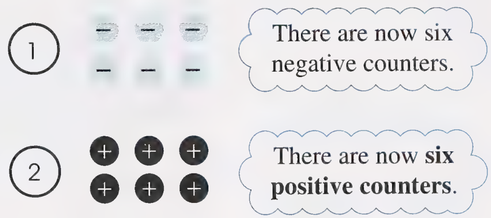
Use modelling to evaluate the expression $(-2)\times(-3)$.

Solution

Step 1: Model $(+2)\times(+3)$.



Step 2: There are **two** negative factors in the expression $(-2)\times(-3)$. So, replace the six positive counters with their opposites **twice**.



$\therefore\ (-2)\times(-3)=+6$

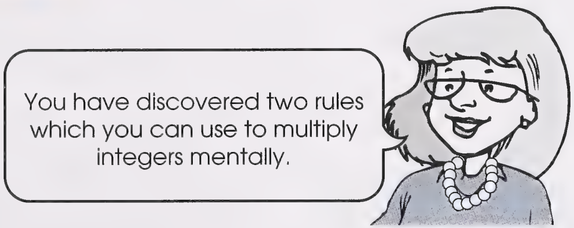
6. **a.** Model the expression $(-3)\times(+4)$; then find the product.

   b. Model the expression $(+5)\times(-5)$; then find the product.

7. **a.** Model the expression $(-3)\times(-4)$; then find the product.

   b. Model the expression $(-5)\times(-5)$; then find the product.

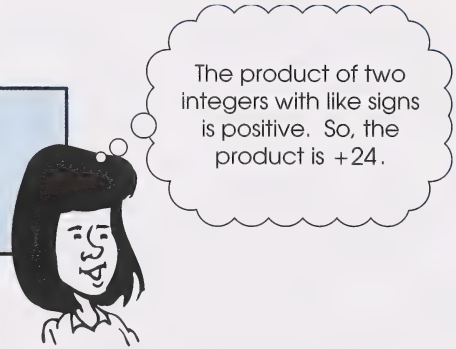
Check your answers by turning to the Appendix.



- The product of two integers with like signs is a positive integer.
- The product of two integers with unlike signs is a negative integer.

Example 10

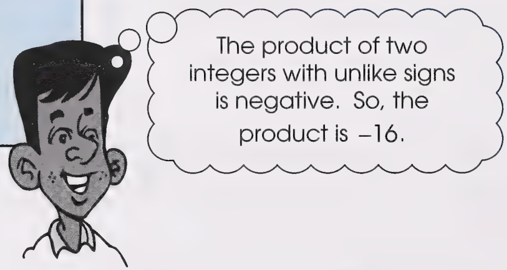
$$\begin{array}{r} (-3) \\ \times\ (-8) \\ \hline \end{array}$$



$\therefore\ (-3)\times(-8)=+24$

Example 11

$(+8)\times(-2)$



$\therefore\ (+8)\times(-2)=-16$

8. State whether each product is positive or negative. Explain why. Then find each of the products.

a. $(-4)\times(+3)$ **b.** $(-4)\times(-4)$ **c.** $(+5)\times(-2)$

d. $\begin{array}{r} (+2) \\ \times\ (-6) \\ \hline \end{array}$ **e.** $\begin{array}{r} (+3) \\ \times\ (+9) \\ \hline \end{array}$ **f.** $\begin{array}{r} (-7) \\ \times\ (-3) \\ \hline \end{array}$

9. Solve the problems given in question 1 of this activity.

Check your answers by turning to the Appendix.

Now Try This

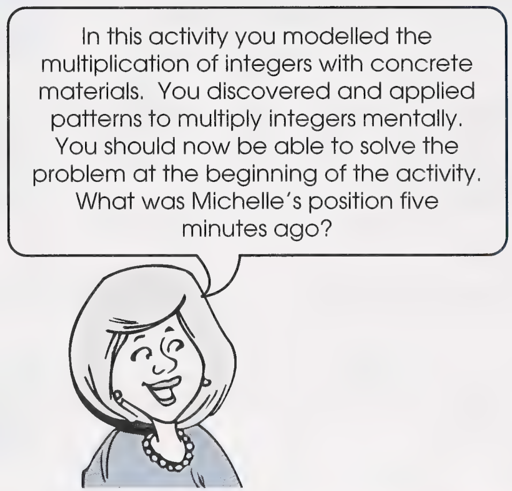
Use a problem-solving strategy to answer the following questions.

10. This figure is formed by nine squares of the same size. If the area of the figure is 144 cm^2, what is its perimeter?

11. If the counting numbers are arranged in four columns, as shown, under which letter will the number 100 appear?

A	B	C	D
1	2	3	4
8	7	6	5
9	10	11	12
16	15	14	13

Check your answers by turning to the Appendix.



Activity 4: Dividing Integers

The present temperature is 0°C. If the temperature is falling 2°C each hour, how long will it take to reach −10°C?

To solve this problem you can write a mathematical expression involving division of integers.

The expression can be written horizontally or vertically.

$$\underbrace{(-10)}_{\text{temperature}} \div \underbrace{(-2)}_{\text{decrease per hour}} \qquad \underbrace{(-2)}_{\text{decrease per hour}}\overline{\big)\underbrace{(-10)}_{\text{temperature}}}$$

1. Write a mathematical expression to describe each of the following situations. (Do not calculate any answers at this point.)

   a. A chinook caused the temperature to rise 10°C in five minutes. What was the change in temperature per minute?

   b. When Fred turned his new freezer on, it took 6 h for the temperature to fall 30°C. What was the change in temperature per hour?

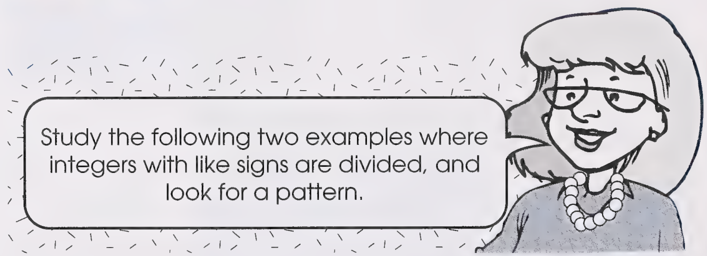
   c. A submarine descends at the rate of 100 m per minute. How many minutes will it take to descend from sea level to 2000 m below sea level?

Check your answers by turning to the Appendix.

Note: You will solve the problems in question 1 at the end of this activity.

To visualize the division of integers, gather your counters and a container, view the segment entitled "Dividing Integers" of the program *Integers* from the series *Math Moves*, and do the video assignments.

Check your answers by viewing the video.



Example 1

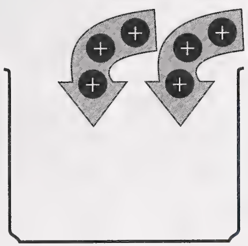
Use modelling to evaluate the expression $(+6) \div (+2)$.

Solution

Step 1: Division and multiplication are related. The equation $(+6) \div (+2) = \square$ means the same as $(+2) \times \square = +6$.

Step 2: Model the equation $(+2) \times \square = +6$, and find the missing factor. **Hint:** Ask yourself this question: To get six positive counters in the container, put in two groups of what?

Start with zero.



Put in two groups of **three positive counters** to get six positive counters in the container.

The missing factor is $+3$.

Step 3: Find the quotient in $(+6) \div (+2) = \square$. **Hint:** The quotient is the same as the missing factor.

$\therefore \ (+6) \div (+2) = +3$

Example 2

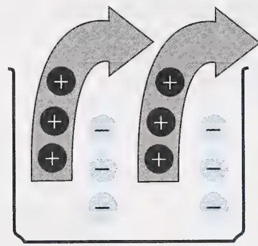
Use modelling to evaluate the expression $(-6) \div (-2)$.

Solution

Step 1: Division and multiplication are related. The equation $(-6) \div (-2) = \square$ means the same as $(-2) \times \square = -6$.

Step 2: Model the equation $(-2) \times \square = -6$, and find the missing factor. **Hint:** Ask yourself this question: To get six negative counters in the container, take out two groups of what?

Start with six zero pairs.



Take out two groups of **three positive counters** to get six negative counters in the container.

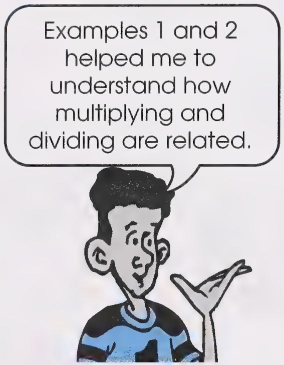
The missing factor is $+3$.

Step 3: Find the quotient in $(-6) \div (-2) = \square$. **Hint:** The quotient is the same as the missing factor.

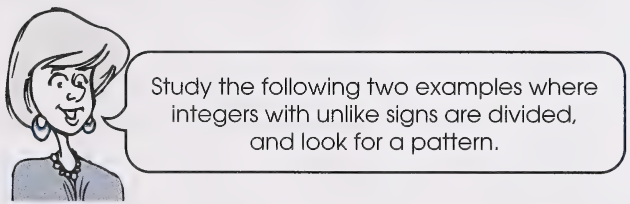
$\therefore \ (-6) \div (-2) = +3$

2. Use Example 1 and Example 2 to answer this question: What do you notice about the sign of the quotient of two integers with like signs?

Check your answer by turning to the Appendix.



You are now ready for the division of integers with unlike signs.



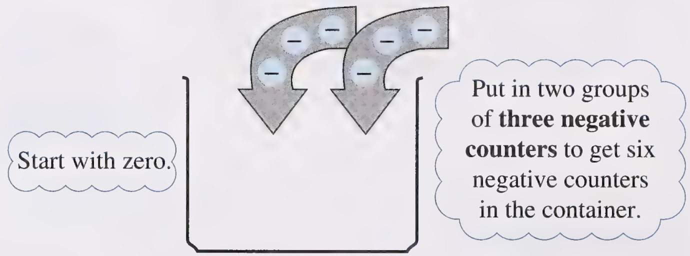
Example 3

Use modelling to evaluate the expression $(-6) \div (+2)$.

Solution

Step 1: Division and multiplication are related. The equation $(-6) \div (+2) = \square$ means the same as $(+2) \times \square = -6$.

Step 2: Model the equation $(+2) \times \square = -6$, and find the missing factor. **Hint:** Ask yourself this question: To get six negative counters in the container, put in two groups of what?



The missing factor is -3.

Step 3: Find the quotient in $(-6) \div (+2) = \square$. **Hint:** The quotient is the same as the missing factor.

$\therefore\ (-6) \div (+2) = -3$

Example 4

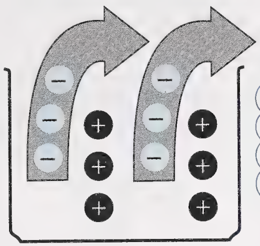
Use modelling to evaluate the expression $(+6) \div (-2)$.

Solution

Step 1: Division and multiplication are related. The equation

$(+6) \div (-2) = \square$ means the same as $(-2) \times \square = +6$.

Step 2: Model the equation $(-2) \times \square = +6$, and find the missing factor. **Hint:** Ask yourself this question: To get six positive counters in the container, take out two groups of what?



The missing factor is -3.

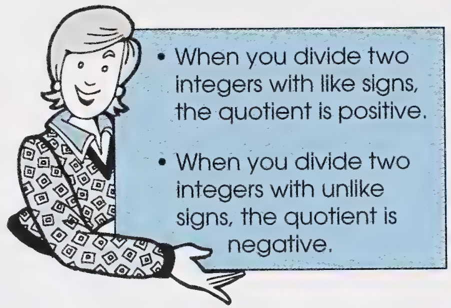
Step 3: Find the quotient in $(+6) \div (-2) = \square$. **Hint:** The quotient is the same as the missing factor.

$\therefore\ (+6) \div (-2) = -3$

3. Use Example 3 and Example 4 to answer this question: What do you notice about the sign of the quotient of two integers with unlike signs?

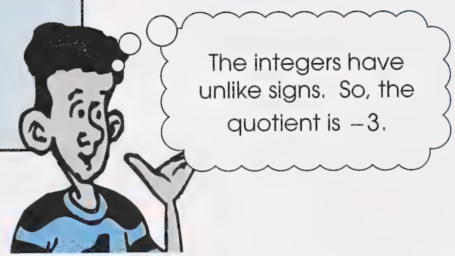
Check your answer by turning to the Appendix.

You have discovered two rules that will help you divide integers mentally.



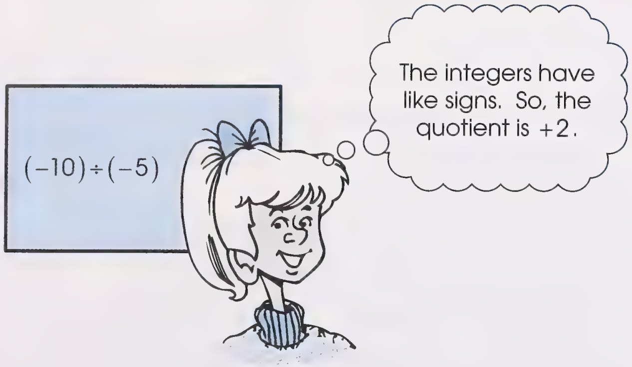
Example 5

$(+5)\overline{)(-15)}$



$\therefore\ (-15) \div (+5) = -3$

Example 6



$\therefore \ (-10) \div (-5) = +2$

4. State whether each quotient is positive or negative. Explain why. Then find each quotient.

a. $(-8) \div (-2)$ **b.** $(-18) \div (+2)$

c. $(-3)\overline{)(-15)}$ **d.** $(+5)\overline{)(-10)}$

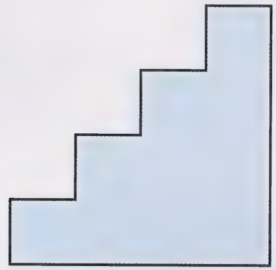
5. Solve the problems given in question 1 of this activity.

Check your answers by turning to the Appendix.

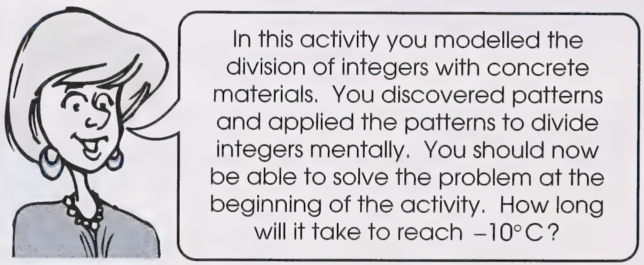
Now Try This

Use a problem-solving strategy to answer the following question.

6. Find the area of the given figure. The line segments for the step formation are 3 units wide and 3 units deep.



Check your answer by turning to the Appendix.



Activity 5: Order of Operations

WESTFILE INC.

As a child, you discovered that many tasks need to be done in a certain order. For example, you put on your socks before you put on your shoes.

When you calculate a series of operations, you also follow a certain order—the order of operations.

You studied the rules for the order of operations in Module 4. In this activity you will apply these rules with integers.

Example 1

Evaluate the expression $(-2)+(+3)\times(-4)$.

Solution

$(-2)+(+3)\times(-4)$ ← Multiply.

$= (-2)+(-12)$ ← Add.

$= -14$

Example 2

Evaluate the expression $(-5)\times(-3)\div(+5)\times(-2)$.

Solution

$(-5)\times(-3)\div(+5)\times(-2)$ ← Multiply.

$= (+15)\div(+5)\times(-2)$ ← Divide.

$= (+3)\times(-2)$ ← Multiply.

$= -6$

Example 3

Evaluate the expression $(-5)+(-3)-(-2)$.

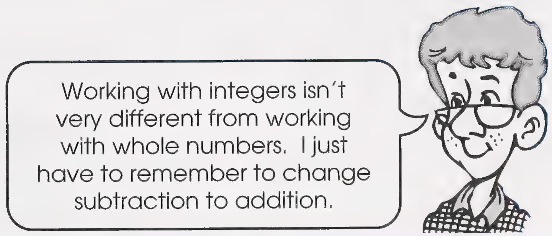
Solution

$(-5)+(-3)-(-2)$ ← Change subtraction to addition.

$= (-5)+(-3)+(+2)$ ← Add.

$= (-8)+(+2)$ ← Add.

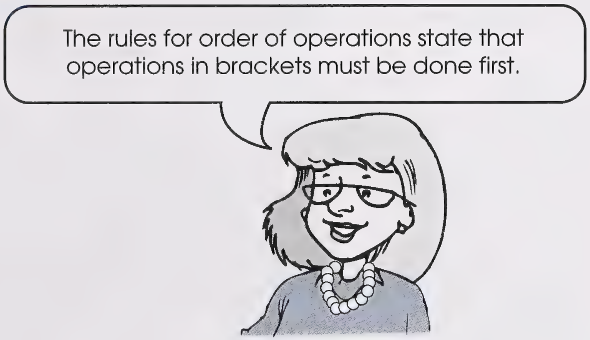
$= -6$



1. Evaluate each of the following mathematical expressions.

   a. $(-2)\times(-6)\div(-3)$
   b. $(+4)+(-2)\times(-5)$
   c. $(-3)-(-4)+(-2)$
   d. $(+14)\div(-2)+(+6)\times(-3)$
   e. $(-2)\times(+3)-(-2)$
   f. $(+7)-(+8)\times(-3)$
   g. $(-4)-(+4)\div(-2)$
   h. $(+9)\div(-3)+(+5)\times(-2)-(+1)$

Check your answers by turning to the Appendix.



In the following examples, two types of brackets are used: parentheses and square brackets. Parentheses are used to emphasize the integers, and square brackets are used to show the order of operations.

Example 4

Evaluate the expression $(+3)\times[(-2)-(-1)]$.

Solution

$(+3)\times[(-2)-(-1)]$ ← Work in square brackets first; change subtraction to addition.

$= (+3)\times[(-2)+(+1)]$ ← Add.

$= (+3)\times(-1)$ ← Multiply.

$= -3$

Example 5

Evaluate the expression $(-3)-[(+4)+(-2)]$.

Solution

$(-3)-[(+4)+(-2)]$ ← Work in square brackets first; add.

$= (-3)-(+2)$ ← Change subtraction to addition.

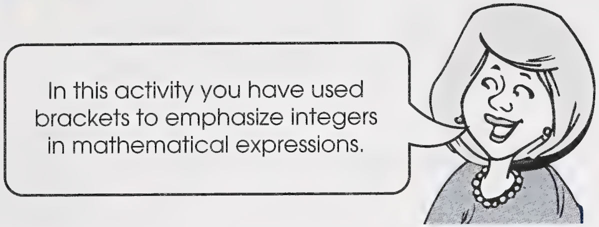
$= (-3)+(-2)$ ← Add.

$= -5$

2. Evaluate each of the following mathematical expressions.

   a. $(-6) \div [(+2)-(-1)]$ b. $(-2)-[(+6)+(-3)]$

Check your answers by turning to the Appendix.



Sometimes the following rules are used to simplify mathematical expressions involving integers.

- The sign of a positive integer is not always written. For example, the expression $(+3)+(+8)$ may be written as $3+8$, and the expression $(+5)-(+8)$ may be written as $5-8$.
- Brackets are always used to separate an operation and a negative integer or to show the order of operations. However, brackets are not always used to emphasize an integer. For example, the expression $(-6)+(3)$ may be written as $-6+3$, the expression $(+2)+(-3)$ may be written as $2+(-3)$, and the expression $(-7)-[(+3)-(+2)]$ may be written as $-7-(3-2)$.

3. Simplify each of the following mathematical expressions by removing the sign of positive integers and brackets that are not required.

   a. $(+5)+(+6)$ b. $(-3)+(+4)$ c. $(-3)+(-2)$
   d. $(-2)-(+5)$ e. $(+3)-(+10)$ f. $(-5)-(-10)$
   g. $(-8)\times(+12)$ h. $(+3)\times(-2)$ i. $(-2)\times(+5)$
   j. $(+6)\div(-2)$ k. $(+30)\div(+5)$ l. $(-25)\div(-5)$
   m. $(-6)\times[(+2)-(-1)]$ n. $(-2)-[(+6)+(-3)]$

Check your answers by turning to the Appendix.

You may find it helpful to rewrite mathematical expressions by placing brackets around positive and negative integers. For example, you may rewrite the expression $2-5$ as $(+2)-(+5)$.

4. Rewrite each of the following expressions with signs and brackets to emphasize the integers.

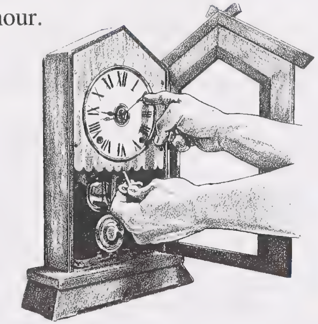
   a. $-6+9$ b. $-4-(-6)$ c. -4×3
   d. $8\div(-4)$ e. $-30\div6$ f. $2-9$
   g. $-5\div(2-3)$ h. $8-(4-6)$ i. $5\times(-3+2)$

Check your answers by turning to the Appendix.

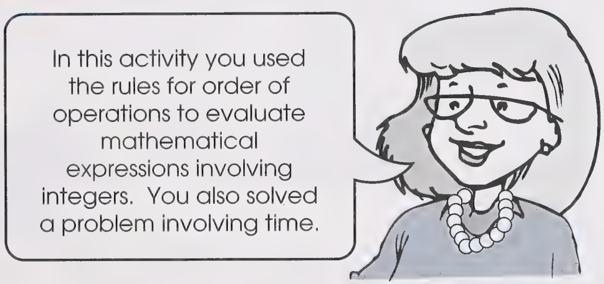
Now Try This

Use a problem-solving strategy to answer the following question.

5. A clock gains five minutes every hour. Frank sets the clock to the correct time at 9:00. If the clock is not reset, when will the clock give the correct time again?



Check your answer by turning to the Appendix.



Activity 6: Using a Calculator

Scientific calculators are very helpful—especially when you are working with very large or very small numbers.

In this activity you will use a calculator to evaluate expressions involving integers.

Many scientific calculators have a **sign-change key**, which looks like $\boxed{+/-}$. The key is used to change the sign of a number.

You do **not** need to use the sign-change key to enter a positive number.

- To enter $+8$, press $\boxed{8}$.
- To enter $+95$, press $\boxed{9}\,\boxed{5}$.

You need to use the sign-change key to enter a negative integer.

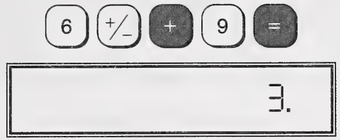
- To enter -8, press $\boxed{8}\,\boxed{+/-}$.
- To enter -95, press $\boxed{9}\,\boxed{5}\,\boxed{+/-}$.

The following examples show how you can use a calculator to perform operations on integers.

Example 1

Evaluate the expression $(-6)+(+9)$.

Solution

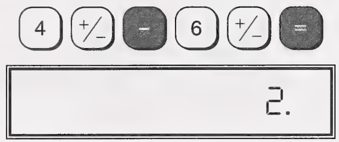


$\therefore\ (-6)+(+9)=+3$

Example 2

Evaluate the expression $(-4)-(-6)$.

Solution

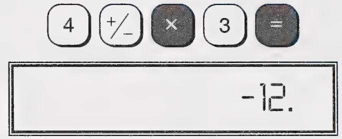


$\therefore\ (-4)-(-6)=+2$

Example 3

Evaluate the expression $(-4)\times(+3)$.

Solution

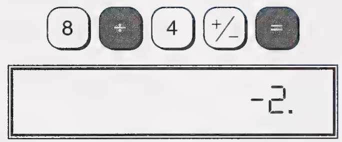


$\therefore\ (-4)\times(+3)=-12$

Example 4

Evaluate the expression $(+8)\div(-4)$.

Solution



$\therefore\ (+8)\div(-4)=-2$

Use a scientific calculator with a sign-change key to answer question 1.

1. Evaluate each of the following expressions.

a. $(-27)+(+86)$ b. $(-132)-(-56)$

c. $(-25)\times(-15)$ d. $(-245)\div(+35)$

Check your answers by turning to the Appendix.

Some scientific calculators have **bracket keys**, which look like (and). The bracket keys are used to show the order of operations.

Example 5

Evaluate the expression $(+3)\times[(-2)-(-3)]$. **Hint:** Use the bracket keys around $(-2)-(-3)$ to show that, in this expression, subtraction is to be done before multiplication.

Solution

3 × (2 +/− − 3 +/−) =

3.

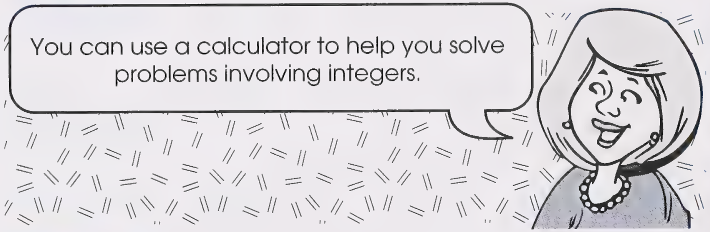
Use a scientific calculator with bracket keys to answer question 2.

2. Evaluate each of the following mathematical expressions.

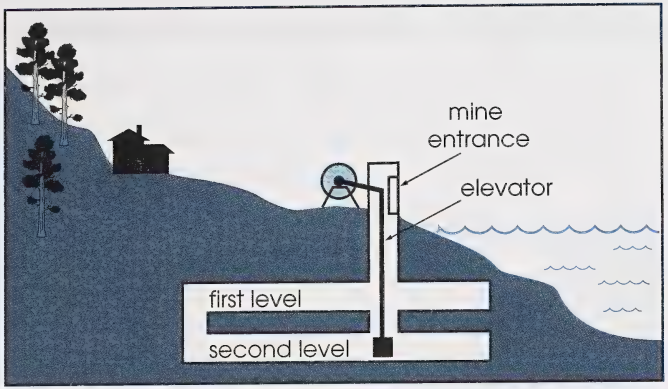
a. $[(-150)+(+6)]\div(+12)$

b. $(+13)-[(-2)+(-10)]$

Check your answers by turning to the Appendix.

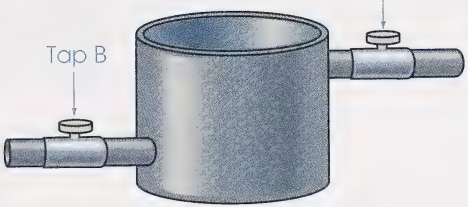


Use a calculator to answer questions 3 and 4.

3. 

The mine entrance in the diagram is 125 m above sea level. The second level of the mine is 500 m below sea level. If the miners travelled in the elevator from the mine entrance to the second level, how far did they rise or descend?

4. In this storage tank, water flows in by opening Tap A. Water flows out by opening Tap B.



Calculate the total change in the volume of water in the tank if Tap A is closed and Tap B is opened for 15 minutes, and water is flowing out at a rate of 13 L per minute.

5. The greatest temperature variation in a 24-h period was recorded on January 23–24, 1916, in Browning, Montana, U.S.A. If the high temperature on January 23 was 7°C and the low temperature on January 24 was -49°C, what was the temperature change?

Check your answers by turning to the Appendix.

Now Try This

Use a problem-solving strategy to answer the following question.

6. Figure A and Figure B are each made up of nine identical squares. If the perimeter of Figure A is 48 units, what is the perimeter of Figure B?

Figure A

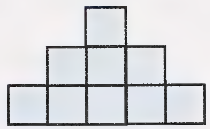
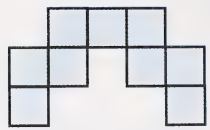


Figure B



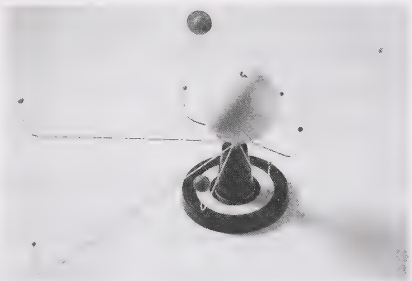
Check your answer by turning to the Appendix.

Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

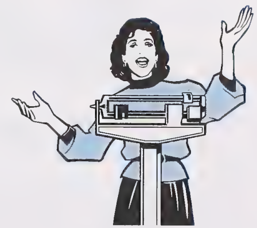
Have you ever used a model of the Sun and the planets in the solar system? A model like the one in the photograph is helpful in comparing the sizes of the planets and the Sun. It is also helpful in showing the positions of the planets and how they orbit the Sun.



SPECTRUM

Models are also helpful in visualizing mathematical concepts. In this activity you will use number lines to model operations on integers.

Example 1

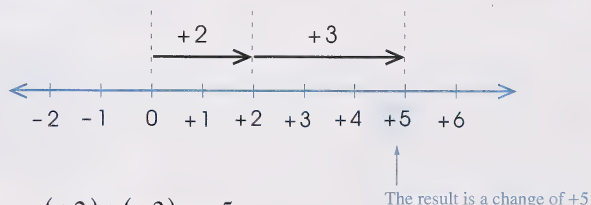


Susan is body-building. During the first year of training, she gained 2 kg. During the second year of training, she gained another 3 kg. How much did she gain altogether in the two years?

Solution

These events can be described by the mathematical expression $(+2)+(+3)$.

The following number line may help you picture the events.



$\therefore\ (+2)+(+3)=+5$

Susan gained 5 kg altogether.

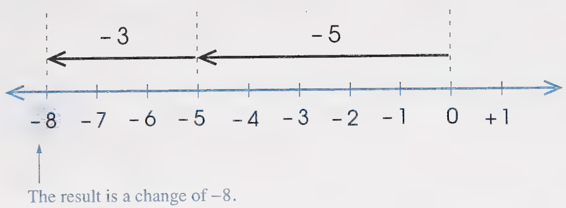
Example 2

While Charles and Myrtle were sailing on the lake, a temperature drop of 5°C was followed by another drop of 3°C. What was the total change in temperature?

Solution

These events can be described by the mathematical expression $(-5)+(-3)$.

The following number line may help you picture the events.



$\therefore\ (-5)+(-3)=-8$

The total change was a drop of 8°C.

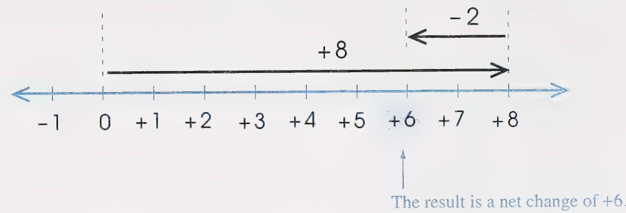
Example 3

A football team gained eight yards on one play and then lost two yards on the next play. In total, how many yards were gained or lost?

Solution

These events can be described by the mathematical expression $(+8)+(-2)$.

The following number line may help you picture the events.



$\therefore\ (+8)+(-2)=+6$

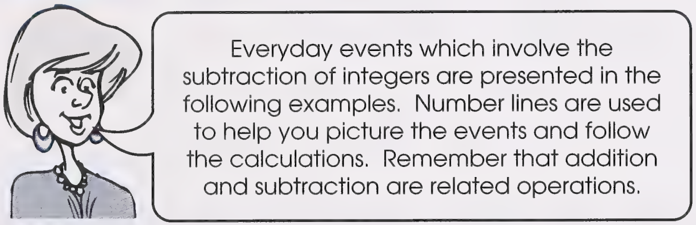
In total, there was a gain of 6 yards.

1. Use number lines to evaluate each mathematical expression.

a. $(+2)+(+5)$ b. $(-3)+(-5)$ c. $(-3)+(+6)$

d. $(+3)+(-4)$ e. $(-2)+(+1)$ f. $(+2)+(-3)$

Check your answers by turning to the Appendix.



Example 4

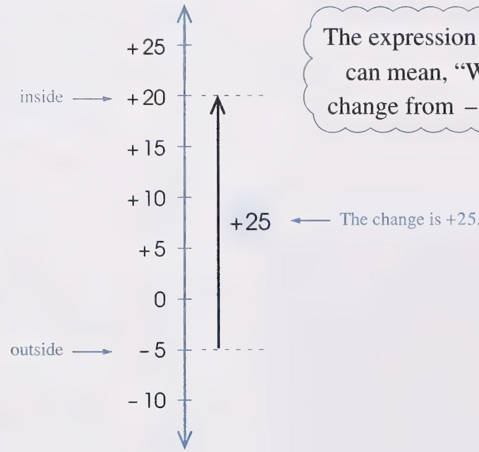
On a winter day the temperature inside a house was $+20\,°C$, while the temperature outside was $-5\,°C$. What was the temperature change from the outside to the inside of the house?

Solution

The situation can be described by the following mathematical sentences.

$$\underbrace{(+20)}_{\text{inside}}-\underbrace{(-5)}_{\text{outside}}=\square \qquad \textbf{or} \qquad \underbrace{(-5)}_{\text{outside}}+\square=\underbrace{+20}_{\text{inside}}$$

The following number line may help you picture the situation.



The expression $+20-(-5)$ can mean, "What is the change from -5 to $+20$?"

$\therefore\ (+20)-(-5)=+25$

The change in temperature from the outside to the inside of the house was $+25\,°C$.

Example 5

The following chart shows the opening and closing share prices for Allison Products. What was the change in the share price?

Company	Opening Price ($)	Closing Price ($)
Allison Products	5	2

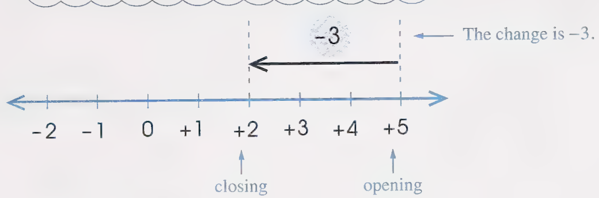
Solution

The situation can be described by these mathematical sentences.

$$\underset{\text{closing}}{(+2)} - \underset{\text{opening}}{(+5)} = \square \quad \textbf{or} \quad \underset{\text{opening}}{(+5)} + \square = \underset{\text{closing}}{+2}$$

The following number line may help you picture the situation.

The expression $(+2)-(+5)$ can mean, "What is the change from $+5$ to $+2$?"



$\therefore\ (+2)-(+5) = -3$

The change is a loss of $3 per share.

Example 6

The following chart shows the opening and closing share prices for Beverly Ltd. What was the change in the share price?

Company	Opening Price ($)	Closing Price ($)
Beverly Ltd.	7	9

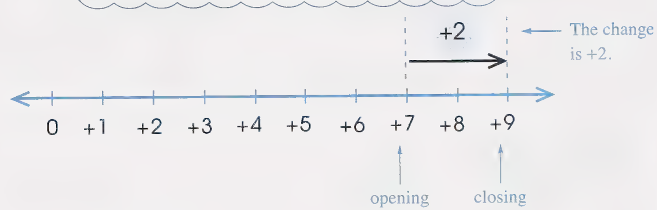
Solution

The situation can be described by these mathematical sentences.

$$\underset{\text{closing}}{(+9)} - \underset{\text{opening}}{(+7)} = \square \quad \textbf{or} \quad \underset{\text{opening}}{(+7)} + \square = \underset{\text{closing}}{+9}$$

The following number line may help you picture the situation.

The expression $(+9)-(+7)$ can mean, "What is the change from $+7$ to $+9$?"



$\therefore\ (+9)-(+7) = +2$

The change is a gain of $2 per share.

2. Use a number line to evaluate each of the following mathematical expressions.

a. $(+6)-(+3)$	**b.** $(-6)-(-5)$
c. $(-5)-(-6)$	**d.** $(+3)-(-3)$
e. $(-7)-(+2)$	**f.** $(+2)-(-5)$

3. Use number lines to evaluate each pair of mathematical expressions.

a. $(+1)-(+3)$; $(+1)+(-3)$

b. $(-2)-(-1)$; $(-2)+(+1)$

4. **a.** What did you notice about the sums and differences in question 3?

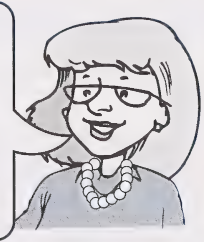
b. How is knowing this relationship helpful?

Check your answers by turning to the Appendix.

There is a page of Droopy[1] cut-outs in the Appendix. Photocopy the page, glue the learning aids to heavy paper, and cut them out.

[1] National Council of Teachers of Mathematics for Droopy learning aids from *Arithmetic Teacher*, December 1976, Reston, Virginia.

Notice that one Droopy has a positive sweater and the other Droopy has a negative sweater. You can use a Droopy and a number line to help you find the product of two integers. Remember, multiplication is repeated addition.



Droopy tackles the multiplication of two integers by using the following rules.

- Droopy always starts at zero on the number line.
- The sign of the first factor determines which sweater Droopy will wear and the direction he will face. If the sign of the first factor is positive, Droopy wears his positive sweater and faces the positive direction. If the sign of the first factor is negative, Droopy wears his negative sweater and faces the negative direction.
- The sign of the second factor determines which way Droopy moves. If the sign of the second factor is positive, Droopy moves forward. If the sign of the second factor is negative, he moves backward.
- The absolute value of the first factor determines the number of steps.
- The absolute value of the second factor determines the size of each step.
- The product is the integer where Droopy stops.

Example 7

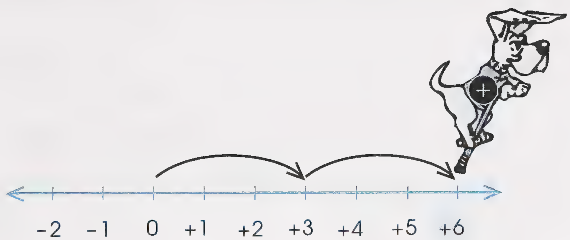
Evaluate the mathematical expression $(+2)\times(+3)$.

Solution

Step 1: Use this reasoning.

- The first factor is positive. So, Droopy will wear his positive sweater and face the positive direction.
- The second factor is positive. So, Droopy will move forward.
- The absolute value of $+2$ is 2. The absolute value of $+3$ is 3. So, Droopy will take two steps of 3.

Step 2: Use Droopy and a number line to find the product. Ask yourself this question: Where does Droopy stop?



Droopy stops at $+6$.

$\therefore\ (+2)\times(+3)=+6$

Example 8

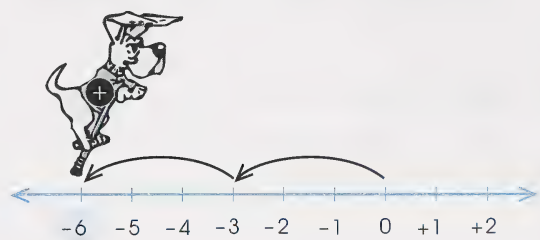
Evaluate the mathematical expression $(+2)\times(-3)$.

Solution

Step 1: Use this reasoning.

- The first factor is positive. So, Droopy will wear his positive sweater and face the positive direction.
- The second factor is negative. So, Droopy will move backwards.
- The absolute value of $+2$ is 2. The absolute value of -3 is 3. So, Droopy will take two steps of 3.

Step 2: Use Droopy and a number line to find the product. Ask yourself this question: Where does Droopy stop?



Droopy stops at -6.

$\therefore\ (+2)\times(-3)=-6$

Example 9

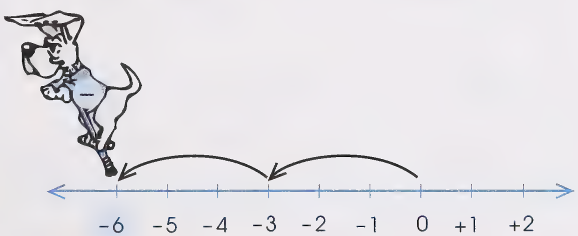
Evaluate the mathematical expression $(-2)\times(+3)$.

Solution

Step 1: Use this reasoning.

- The first factor is negative. So, Droopy will wear his negative sweater and face the negative direction.
- The second factor is positive. So, Droopy will move forward.
- The absolute value of -2 is 2. The absolute value of $+3$ is 3. So, Droopy will take two steps of 3.

Step 2: Use Droopy and a number line to find the product. Ask yourself this question: Where does Droopy stop?



Droopy stops at -6.

$\therefore\ (-2)\times(+3)=-6$

Example 10

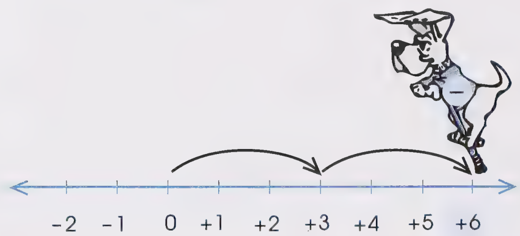
Evaluate the mathematical expression $(-2)\times(-3)$.

Solution

Step 1: Use this reasoning.

- The first factor is negative. So, Droopy will wear his negative sweater and face the negative direction.
- The second factor is negative. So, Droopy will move backwards.
- The absolute value of -2 is 2. The absolute value of -3 is 3. So, Droopy will take two steps of 3.

Step 2: Use Droopy and a number line to find the product. Ask yourself this question: Where does Droopy stop?



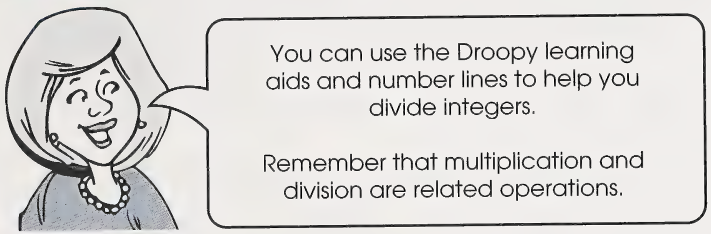
Droopy stops at $+6$.

$\therefore\ (-2)\times(-3)=+6$

5. Use the cut-out Droopy learning aids from the Appendix and a number line to model each of the following expressions and find the product.

a. $(+2)\times(+4)$ **b.** $(+2)\times(-4)$
c. $(-2)\times(+4)$ **d.** $(-2)\times(-4)$

Check your answers by turning to the Appendix.



Example 11

Evaluate the mathematical expression $(+6)\div(+2)$.

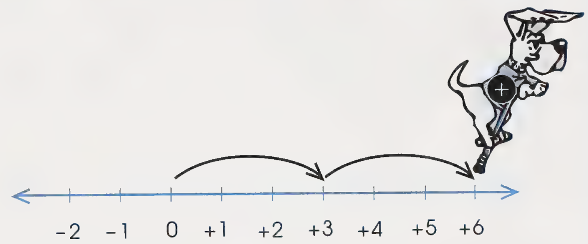
Solution

The equation $(+6)\div(+2)=$ means the same as $(+2)\times$ $=+6$.

Step 1: Use this reasoning.

- The first factor is positive. So, Droopy will wear his positive sweater and face the positive direction.
- The product is $+6$. So, Droopy will stop at $+6$.
- The absolute value of $+2$ is 2. So, Droopy will take two steps.

Step 2: Use Droopy and a number line to find the missing factor. Ask yourself this question: To get to $+6$, in what direction will Droopy move and what will be the size of each step?



In order for Droopy to get to $+6$, he must move **forward** in two steps of **3**. So, the missing factor is $+3$.

Step 3: Find the quotient.

The quotient in $(+6)\div(+2)=$ is the same as the missing factor.

$\therefore\ (+6)\div(+2)=+3$

Example 12

Evaluate the mathematical expression $(-6)\div(+2)$.

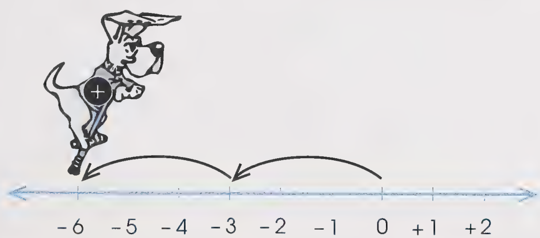
Solution

The equation $(-6)\div(+2)=$ means the same as $(+2)\times$ $=-6$.

Step 1: Use this reasoning.

- The first factor is positive. So, Droopy will wear his positive sweater and face the positive direction.
- The product is -6. So, Droopy will stop at -6.
- The absolute value of $+2$ is 2. So, Droopy will take two steps.

Step 2: Use Droopy and a number line to find the missing factor. Ask yourself this question: To get to -6, in what direction will Droopy move and what will be the size of each step?



In order for Droopy to get to -6, he must move **backwards** in two steps of **3**. So, the missing factor is -3.

Step 3: Find the quotient.

The quotient in $(-6)\div(+2)=$ is the same as the missing factor.

$\therefore\ (-6)\div(+2)=-3$

Example 13

Evaluate the mathematical expression $(-6)\div(-2)$.

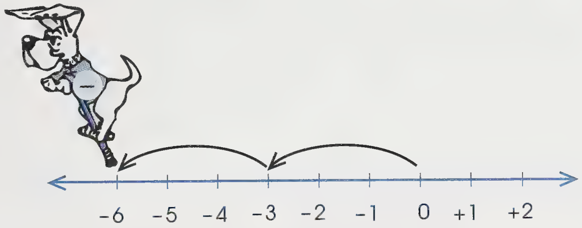
Solution

The equation $(-6)\div(-2)=$ means the same as $(-2)\times$ $=-6$.

Step 1: Use this reasoning.

- The first factor is negative. So, Droopy will wear his negative sweater and face the negative direction.
- The product is -6. So, Droopy will stop at -6.
- The absolute value of -2 is 2. So, Droopy will take two steps.

Step 2: Use Droopy and a number line to find the missing factor. Ask yourself this question: To get to -6, in what direction will Droopy move and what will be the size of each step?



In order for Droopy to get to -6, he must move **forward** two steps of **3**. So, the missing factor is $+3$.

Step 3: Find the quotient.

The quotient in $(-6) \div (-2) = \square$ is the same as the missing factor.

$\therefore \ (-6) \div (-2) = +3$

Example 14

Evaluate the mathematical expression $(+6) \div (-2)$.

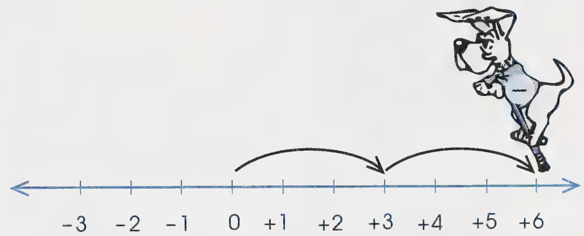
Solution

The equation $(+6) \div (-2) = \square$ means the same as $(-2) \times \square = +6$.

Step 1: Use this reasoning.

- The first factor is negative. So, Droopy will wear his negative sweater and face the negative direction.
- The product is $+6$. So, Droopy will stop at $+6$.
- The absolute value of -2 is 2. So, Droopy will take two steps.

Step 2: Use Droopy and a number line to find the missing factor. Ask yourself this question: To get to $+6$, in what direction will Droopy move and what will be the size of each step?



In order for Droopy to get to $+6$, he must move **backwards** two steps of **3.** So, the missing factor is -3.

Step 3: Find the quotient.

The quotient in $(+6) \div (-2) = \square$ is the same as the missing factor.

$\therefore \ (+6) \div (-2) = -3$

6. Use the cut-out Droopy learning aids from the Appendix and number lines to evaluate the following mathematical expressions.

a. $(+6) \div (+3)$ b. $(+8) \div (-2)$
c. $(-8) \div (+4)$ d. $(-8) \div (-2)$

Check your answers by turning to the Appendix.

Enrichment

Ruth plans to visit a friend. She looks at the thermometer and notices that the temperature is $-10\,°C$.

When she goes outside, she discovers that there is a strong wind. The combined effect of the wind and the air temperature makes it seem much colder than $-10\,°C$.

When Ruth arrives at her friend's house, the radio is playing. The announcer says the **wind-chill equivalent temperature** is $-24\,°C$!

The wind-chill equivalent temperature is an indication of how cold the wind makes the air feel.

The following table can be used to calculate the wind-chill equivalent temperature.

Wind-Chill Equivalent Temperature (°C)

Wind Speed (km/h)	Actual Thermometer Reading (°C)							
	0	−5	−10	−15	−20	−25	−30	−35
10	−2	−7	−12	−17	−22	−27	−32	−38
20	−7	−13	−19	−25	−31	−37	−43	−50
30	−11	−17	−24	−31	−37	−44	−50	−57
40	−13	−20	−27	−34	−41	−48	−55	−62
50	−15	−22	−29	−36	−44	−51	−58	−66
60	−16	−23	−31	−38	−45	−53	−60	−68

Use the preceding table to answer questions 1 to 4.

1. If the wind is blowing at about 20 km/h, estimate the wind-chill equivalent temperature for each of the following actual thermometer readings.

   a. $-10\,°C$ b. $-15\,°C$ c. $-20\,°C$ d. $-25\,°C$

2. Calculate how much colder the wind-chill equivalent temperature is than the actual thermometer reading for each of the temperatures in question 1.

3. During a five-hour period, the thermometer reading dropped from $-5\,°C$ to $-20\,°C$. The wind remained at a constant speed of 30 km/h during that period.

   a. What are the corresponding wind-chill equivalent temperatures?

b. Which decreased more, the actual thermometer readings or the wind-chill equivalent temperatures?

4. Travelling at a certain speed produces the same effect as the wind blowing at that speed. If you were driving a snowmobile at 40 km/h and the thermometer read $-10\,°C$, what wind-chill equivalent temperature would you experience?

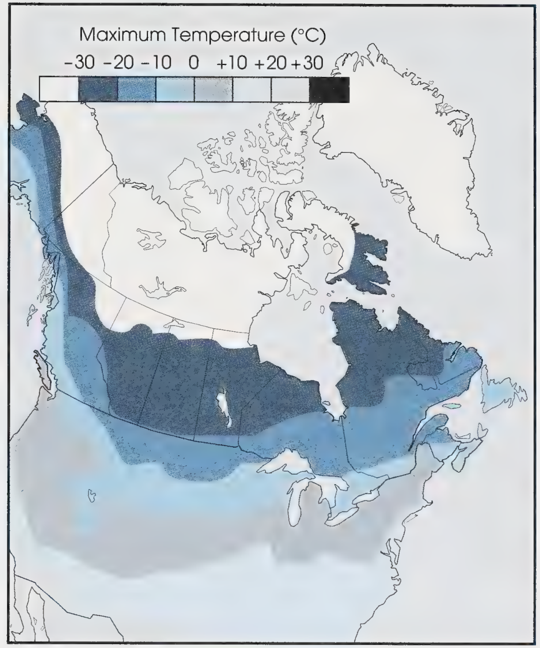
Check your answers by turning to the Appendix.

Use the Internet to discover more about wind-chill equivalent temperatures. Here is the uniform resource locator (URL) of an interesting site.

http://www.on.doe.ca/comm/windchil.htm

Did You Know?

The weather section of some newspapers includes maps like the following.



Notice that areas of similar temperature are coloured or shaded to help the readers visualize temperature patterns across North America.

Do you think this map is showing temperatures in summer or winter? Why?

Conclusion

WESTFILE INC.

In this section you modelled the operation of integers with concrete materials and performed operations mentally and with a calculator.

During the 1990–91 hockey season, the Edmonton Oilers scored 272 goals. They had 272 goals scored against them. Using the rating system described at the beginning of this section, Joe Murphy had a plus-minus figure of $+2$. His teammate, Adam Graves, had a plus-minus figure of -21.

To calculate plus-minus figures, you must be able to perform operations on integers.

Assignment

Assignment Booklet

You are now ready to complete the module assignment for Section 2.

Section 3: Using Integers in Algebra

NASA

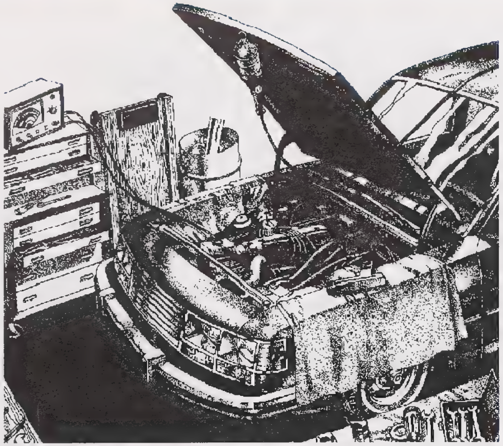
This photograph of Earth was taken from Apollo 17, a spacecraft launched in December 1972. Earth is clearly seen as a ball-like shape in space.

People did not always believe Earth was round. Before Christopher Columbus travelled to America, many sailors were afraid to make the voyage across the ocean. Because they believed Earth was flat, they were afraid that if they sailed too far, they might fall off the edge.

Early mathematicians had similar misconceptions about negative numbers. Diophantus, a Greek mathematician who lived in the third century, called equations like $x+3=1$ "absurd." Today, solving equations like $x+3=1$ is a simple matter; the solution of $x+3=1$ is $x=-2$.

In this section you will use a systematic method to solve equations like $x+3=1$. You will also use integers to evaluate algebraic expressions.

Activity 1: Algebraic Expressions



When a car battery is being charged, the mechanic must be careful to connect the correct wires to the positive terminal and the negative terminal. In order to assist mechanics, manufacturers label battery terminals with a positive sign and a negative sign. The wires on the battery charger are different colours.

In sections 1 and 2, you used two-coloured counters to help you visualize integers.

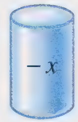
 represents $+3$.

 represents -2.

 represents 0.

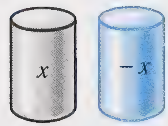
You will continue to use models to help you picture algebraic expressions involving integers. When modelling algebraic expressions, you can use cylinders to represent variables.

represents x.

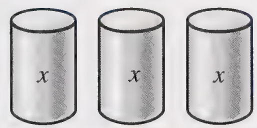


represents $-x$.

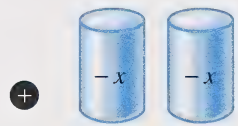
$-x$ means the opposite of x.



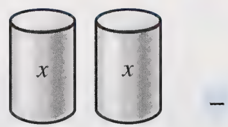
represents 0.



represents $3x$.



represents $1-2x$.



represents $2x-1$.

There are pages of counters and cylinders in the Appendix. Photocopy the pages, glue the pages to heavy paper, and then cut out the learning aids.

1. Use the counters and cylinders to model each of the following algebraic expressions.

a. $5x$	**b.** $-4x$	**c.** $x+3$
d. $x-4$	**e.** $2x+3$	**f.** $2-3x$

Check your answers by turning to the Appendix.

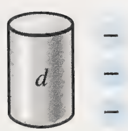
You can evaluate algebraic expressions by replacing the cylinders with counters.

Example 1

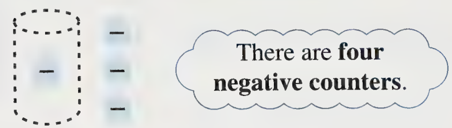
Evaluate $d-3$, if $d=-1$, 0, or $+1$.

Solution

Step 1: Model the algebraic expression.

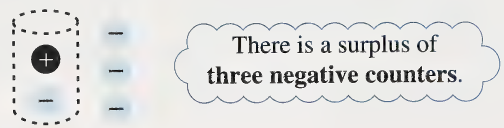


Step 2: Evaluate $d-3$, if $d=-1$. Replace the cylinder with one negative counter and combine the counters.



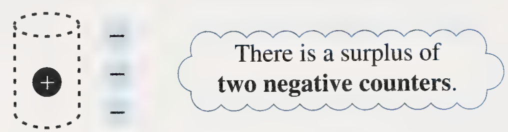
So, if $d=-1$, then $d-3=-4$.

Step 3: To evaluate $d-3$ if $d=0$, replace the cylinder with a zero pair.



So, if $d=0$, then $d-3=-3$.

Step 4: To evaluate $d-3$ if $d=+1$, replace the cylinder with one positive counter.



So, if $d=+1$, then $d-3=-2$.

Use cylinders and counters to answer questions 2 and 3.

2. Evaluate $3x$ for each of the following values of x.

a. $+2$ **b.** -1 **c.** 0

3. Evaluate $2x-1$ for each of the following values of x.

a. $+3$ **b.** -2 **c.** 0

Check your answers by turning to the Appendix.

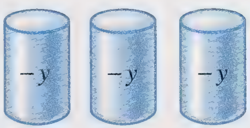
You are now ready to evaluate more algebraic expressions.

Example 2

Evaluate $-3y$, if $y=+2$ or -1.

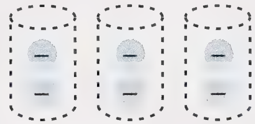
Solution

Step 1: Model the algebraic expression.



Step 2: Use this reasoning: $-y$ is the opposite of y. To evaluate $-3y$, replace each cylinder with the opposite of the given value of y.

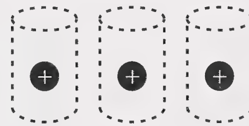
Step 3: Evaluate $-3y$, if $y=+2$. **Hint:** Because -2 is the opposite of $+2$, replace each cylinder with -2.



There are **six negative counters**.

So, if $y=+2$, then $-3y=-6$.

Step 4: Evaluate $-3y$, if $y=-1$. **Hint:** Because $+1$ is the opposite of -1, replace each cylinder with $+1$.



There are **three positive counters**.

So, if $y=-1$, then $-3y=+3$.

Use cylinders and counters to answer questions 4 and 5.

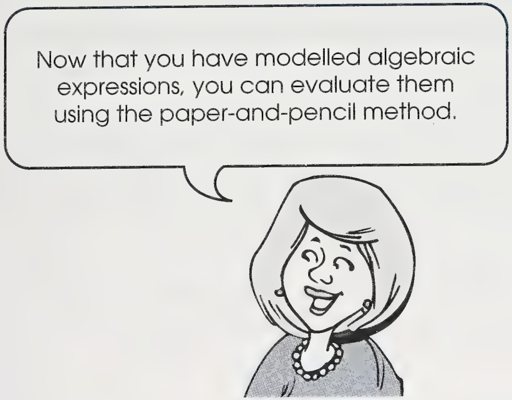
4. Evaluate $2-x$ for each of the following values of x.

a. $+1$ **b.** -2 **c.** 0

5. Evaluate $1-3x$ if x equals each of the following values.

a. $+2$ **b.** -2 **c.** 0

Check your answers by turning to the Appendix.



Example 3

Evaluate $3x-5$, if $x=-2$.

Solution

If $x=-2$, then

Replace x with -2. Then evaluate the expression $3\times(-2)-5$ using the rules for order of operations.

$$
\begin{aligned}
&3x-5\\
&=3\times(-2)-5\\
&=(-6)-5\\
&=(-6)-(+5) \longleftarrow \text{Use parentheses to emphasize the integers.}\\
&=(-6)+(-5) \longleftarrow \text{Change subtraction to addition.}\\
&=-11
\end{aligned}
$$

Example 4

Evaluate $2-5y$, if $y=3$.

Solution

If $y=+3$, then

Replace y with 3. Then evaluate the expression $2-5\times3$ using the rules for order of operations.

$$
\begin{aligned}
&2-5y\\
&=2+(-5)(y)\\
&=2+(-5)(+3)\\
&=(+2)-(+15) \longleftarrow \text{Use parentheses to emphasize the integers.}\\
&=(+2)+(-15) \longleftarrow \text{Change subtraction to addition.}\\
&=-13
\end{aligned}
$$

Use the paper-and-pencil method to answer questions 6 and 7.

6. Evaluate $4a-1$ for each of the following values of a.

a. $a=-3$ **b.** $a=-2$ **c.** $a=-1$ **d.** $a=0$

7. Evaluate $2-5b$ for each of the following values of b.

a. $b=2$ **b.** $b=-3$ **c.** $b=0$ **d.** $b=1$

Check your answers by turning to the Appendix.

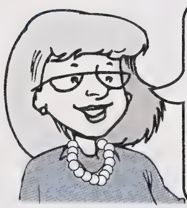
Now Try This

Use a problem-solving strategy to solve the following question.

8. The large rectangle in the diagram is made up of rectangles A, B, C, and D, each of which has whole-number values for the length and width. The area of Rectangle A is 40 square units. The area of Rectangle B is 25 square units. The area of Rectangle C is 30 square units. What is the area of Rectangle D?

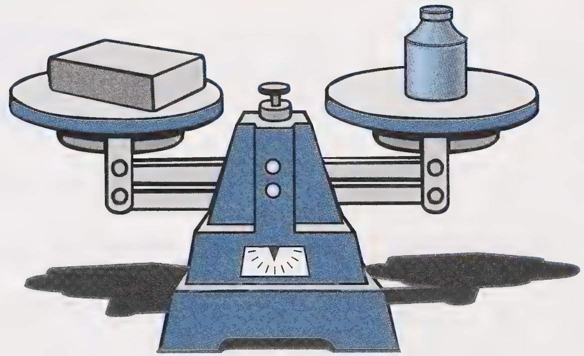
A	*B*
D	*C*

Check your answers by turning to the Appendix.



In this activity, you used an electricity model to help you visualize algebraic expressions involving integers. You evaluated algebraic expressions using both learning aids and the paper-and-pencil method. You continued to solve problems.

Activity 2: Equations



Have you ever used a two-pan balance? The objects on this two-pan balance have equal masses. The needle indicates there is a balance.

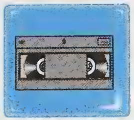
In this activity you will use a balance scale model to help you visualize **equations** involving integers.

An equation is a mathematical statement that contains an equal sign.

There is a balance scale learning aid in the Appendix. Photocopy the page, glue it to a sheet of heavy paper, and cut out this learning aid.

You will use the scale, cylinders, and counters to model and **solve equations**.

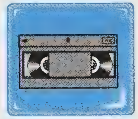
Solving an equation means finding the value of the variable which makes an equation true.



To visualize how to model an equation and solve it by guessing, checking, and revising, gather your learning aids and view the segment entitled "Guess and Test" of the program *Equations–Solving with One Step* from the series *Math Moves* . Do the video assignment.

Check your answers by viewing the video.

You are now ready to use a more systematic method of solving equations.



View the segment entitled "Solving Equations Using Additive Inverses: Equations of Form $x + a = b$" of the program *Equations – Solving with One Step* from the series *Math Moves*. Do the video assignment.

Check your answers by viewing the video.

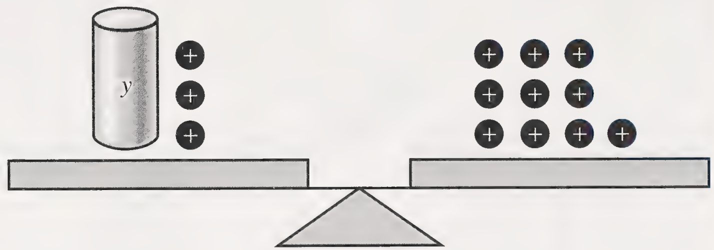
Now that you know how to systematically solve equations of the form $x + a = b$, where x is a variable and a and b are integers, study the following example. Then answer questions 1 and 2.

Example 1

Solve $y + 3 = 10$.

Solution

Step 1: Model the equation.

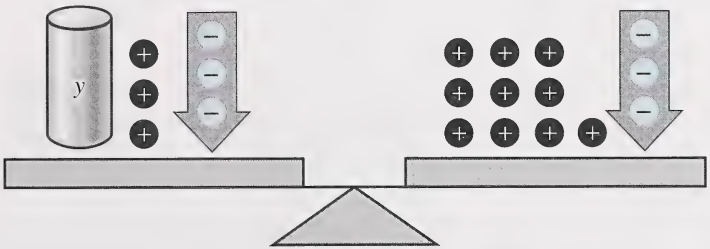


Step 2: Use reasoning to discover how to change $y+3$ on the left side of the equation to y. (This is called **isolating** y.)

$$(+3)+(-3)=0$$

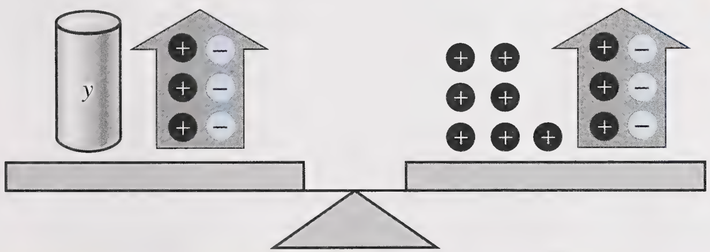
So, you can isolate y by adding -3.

Step 3: Add -3 to the left side of the equation. To maintain equality, add -3 to the right side too.

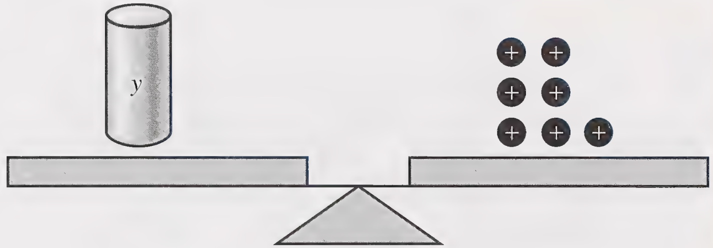


-3 is the additive inverse of $+3$.

Step 4: Simplify the equation by removing the zero pairs. This will not change the balance.



Step 5: Give the solution.



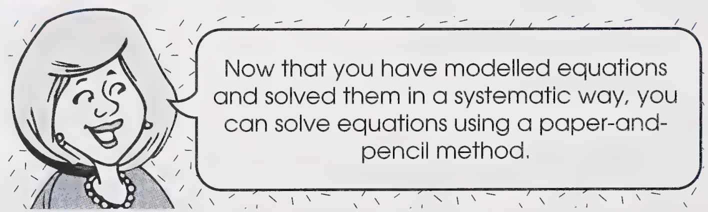
$\therefore\ y=7$

1. **a.** Why was -3 added to the left side of the equation in Step 3 of Example 1?

   b. Why was -3 added to the right side of the equation in Step 3 of Example 1?

2. What rule could you use to solve equations of the form $x+a=b$, where x is a variable and a and b are integers?

Check your answers by turning to the Appendix.



Example 2

Solve $w-3=-2$.

Solution

Method 1: Using Vertical Addition

$$\begin{array}{rr} w\cancel{-3} & =-2 \\ \underline{\cancel{+3}} & \underline{+3} \\ w & =+1 \end{array}$$

In order to isolate w, add $+3$ to each side of the equation.

Method 2: Using Horizontal Addition

$$\begin{aligned} w-3&=-2 \\ w+(-3)&=-2 \\ w+\cancel{(-3)}+\cancel{(+3)}&=(-2)+(+3) \\ w&=+1 \end{aligned}$$

In order to isolate w, add $+3$ to each side of the equation.

Note: Verify the solution.

LS	RS
$w-3$	-2
$=1-3$	
$=(+1)+(-3)$	
$=-2$	

LS = RS

Example 3

Solve $j+4=-6$.

Solution

Method 1: Using Vertical Addition

$$\begin{array}{rr} j\cancel{+4} & =-6 \\ \underline{\cancel{-4}} & \underline{-4} \\ j & =-10 \end{array}$$

In order to isolate j, add -4 to each side of the equation.

Method 2: Using Horizontal Addition

$$\begin{aligned} j+4&=-6 \\ j+(+4)&=-6 \\ j+\cancel{(+4)}+\cancel{(-4)}&=(-6)+(-4) \\ j&=-10 \end{aligned}$$

In order to isolate j, add -4 to each side of the equation.

Note: Verify the solution.

LS	RS
$j+4$	-6
$=-10+4$	
$=(-10)+(+4)$	
$=-6$	

LS = RS

3. What number should be added to both sides in each of the following equations to isolate the variable?

   a. $x+2=7$ b. $m+9=-13$
   c. $t-5=7$ d. $y-2=-8$

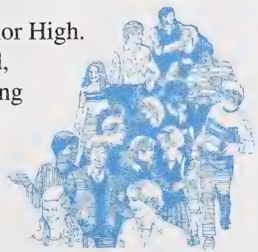
4. Solve each of the equations in question 3 using the paper-and-pencil method. Verify the solutions.

Check your answers by turning to the Appendix.

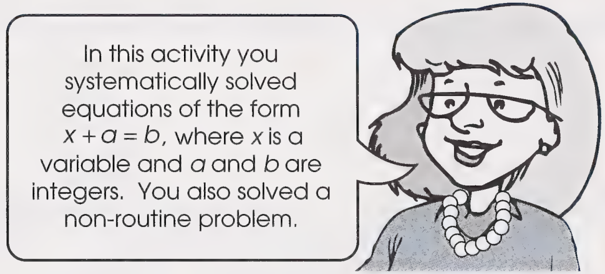
Now Try This

Use a problem-solving strategy to solve the following question.

5. There are 400 students at Parkview Junior High. Of these students, 85 belong to the band, 55 belong to the math club, and 30 belong to both. How many students belong to neither the band nor the math club?



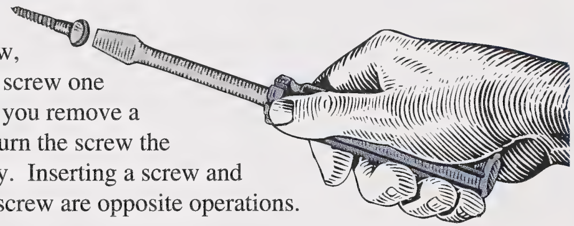
Check your answer by turning to the Appendix.



Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help



When you insert a screw, you turn the screw one way. When you remove a screw, you turn the screw the opposite way. Inserting a screw and removing a screw are opposite operations.

When you solve an equation, you undo the operation that has been done to the variable. If a number has been added to the variable, you can undo this by adding the additive inverse. You can also undo the addition by subtraction. Adding a number and subtracting a number are opposite operations.

To show the relationship between the "doing" operation and the "undoing" operation, you can use a **flow chart** and a **reverse flow chart**.

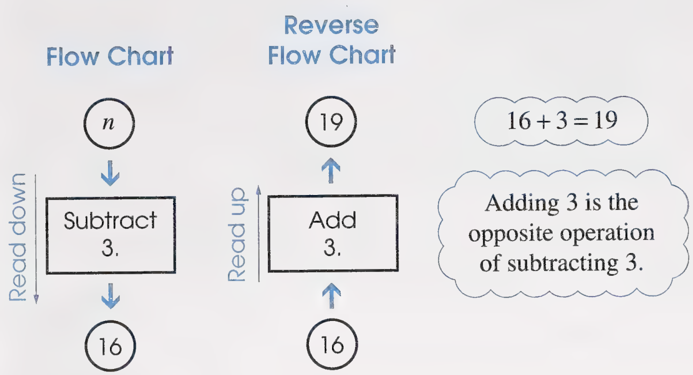
A flow chart is a method for listing steps or instructions. A flow chart starts with a circle, gives steps in instruction boxes, and ends with another circle. A reverse chart undoes the operation set out in a given flow chart.

Example 1

Solve the equation $n - 3 = 16$.

Solution

First, write the equation in flow-chart form. Next, use a reverse flow chart and work backwards to solve the equation.



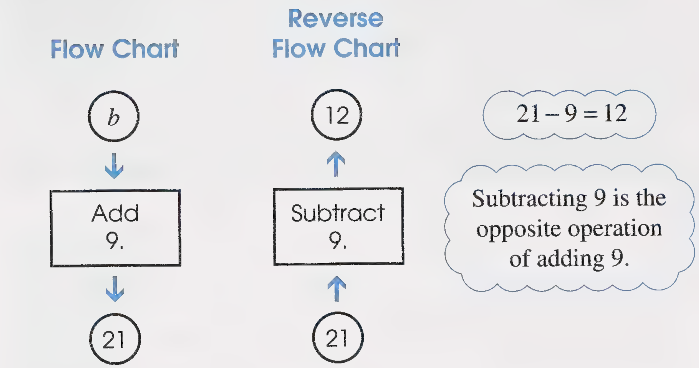
$\therefore\ n = 19$

Example 2

Solve the equation $b + 9 = 21$.

Solution

First, write the equation in a flow chart. Next, use a reverse flow chart and work backwards to solve the equation.



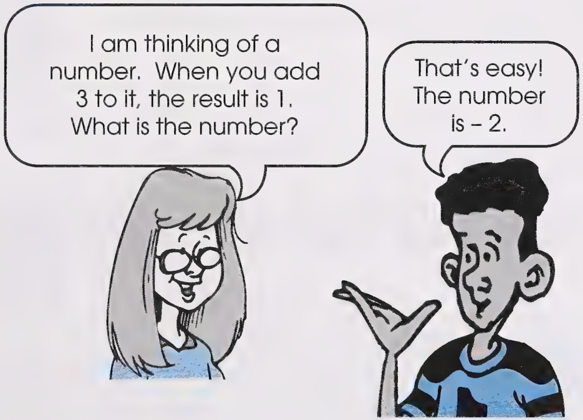
$\therefore\ b = 12$

1. Solve the equation $x + 5 = 8$ using a flow chart and a reverse flow chart.
2. Solve the equation $n - 2 = 10$ using a flow chart and a reverse flow chart.
3. Solve the equation $y + 3 = 1$ using a flow chart and a reverse flow chart.

4. Solve the equation $r - 5 = -2$ using a flow chart and a reverse flow chart.

Check your answers by turning to the Appendix.

Enrichment



Do you like riddles? How did Ahmed solve Linda's riddle?

One way to solve a problem like this is to write an equation and then solve the equation.

Example

Five more than a number is two. Find the number.

Solution

Step 1: Let *n* equal the number, and write an equation.

$$n + 5 = 2$$

Step 2: Solve the equation.

$$\begin{array}{r} n + \cancel{5} = +2 \\ \underline{\cancel{-5} \quad -5} \\ n \quad = -3 \end{array}$$

Add -5 to each side of the equation, and simplify.

So, the number is -3.

Find the puzzle "What Do You Call It When Police Interrogate a Cow's Husband?" in the Appendix. Photocopy the page and then complete the puzzle.

Check your answers by turning to the Appendix.

Conclusion

JIM WHITMER PHOTOGRAPHY

In this section you evaluated algebraic expressions involving integers. You also solved algebraic equations like $x + 3 = 1$.

You discovered that some algebraic equations have solutions which are integers. You may find it humorous to discover that ancient mathematicians once believed that equations like $x + 3 = 1$ were absurd.

Hopefully, the activities in this section made working with variables interesting and fun.

Assignment

Assignment Booklet

You are now ready to complete the module assignment for Section 3.

Module Summary

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In this module you developed a sense for integers. You compared and ordered integers, performed operations on integers, and used integers to evaluate algebraic expressions and solve equations. You continued to solve problems and look for patterns and relationships with numbers.

Games like *Snakes and Ladders*® and *Sorry!*® are fun because sometimes you must go backwards. You can think of moving forward three spaces as $+3$ and going backwards three spaces as -3; $+3$ and -3 are integers.

Integers can be used to describe many other situations which involve direction. Here are a few examples:

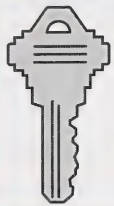
- degrees below freezing; degrees above freezing
- depth below sea level; height above sea level
- golf scores under par; golf scores over par

Final Module Assignment

Assignment Booklet

You are now ready to complete the final module assignment.

APPENDIX



- Glossary
- Suggested Answers
- Cut-out Learning Aids

Glossary

Absolute value: the value of an integer without regard to the sign; also called **magnitude**

Bracket keys: keys on a calculator used to show order of operation

Equation: mathematical statement that contains an equal sign

Flow chart: a method for listing steps or instructions

Integer: any of the numbers in this set: $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

Opposite integer: an integer having the same absolute value but a different sign; also called **additive inverse**

Problem: a task for which the method of finding the answer (as well as the answer) is not immediately known

Reverse flow chart: a flow chart undoes the operations set out in a given flow chart

Sign-change key: key on a calculator used to change the sign of a number

Solving an equation: finding the value of the variables which makes an equation true

Wind-chill equivalent temperature: an indication of how cold the wind makes the air feel

Suggested Answers

Section 1: Activity 1

1. Level 4 $= +4$, Level 3 $= +3$, Level 2 $= +2$, Level 1 $= +1$, Lobby $= 0$, B1 $= -1$, B2 $= -2$, B3 $= -3$

2. **a.** -2 **b.** $+5$

3. **a.** -8 **b.** 0 **c.** $+5$

4. **a.** $+5000$ **b.** -800 **c.** $+20$ **d.** -75
   e. 0 **f.** $+100$ **g.** -45

5. **a.** -36°C, Inuvik **b.** 13°C, Calgary
   c. Halifax **d.** Charlottetown

6. **a.** Windsor Crt, Winspear, World Orgncs, Wld Wide O & G, Yanks Peak, Young-Shann, Yuma Gld, and Zeus Egy Cp stocks went down in value.

   b. Wstn Keltic, Yukon Rev, and Zorah Media stocks showed no changes.

   c. Zappa stock showed the greatest increase; it went up 15¢ per share.

   d. Winspear stock showed the greatest decrease; it went down 15¢ per share.

e. Wld Wide O & G traded the greatest number of shares; it traded 1 080 000 shares. (**Note:** The volume is given in hundreds of shares.)

f. Wstn Logic and Yellowjack were each selling for the least amount at closing; the price was 10¢ per share.

g. Zappa was selling for the greatest amount at closing; the price was $1.60 per share.

7. a. An unnamed point in Antarctica is the lowest point on all the continents.
   b. Mariana Trench is the lowest point in all the oceans.
   c. Mariana Trench is the lowest point on Earth.
   d. Mount Everest is the highest point on Earth.

8. a. $|-3| = 3$ b. $|+7| = 7$ c. $|+5| = 5$ d. $|-4| = 4$

9. a. $+2$ b. -8 c. -6 d. $+3$

Now Try This

10. You can use logic to solve this problem.

- The number is less than $+4$. So, the number could be $+3, +2, +1, 0, -1, -2, -3, -4, \ldots$.
- It is greater than -2. This eliminates $-2, -3, -4, \ldots$. So, the number could be $+3, +2, +1, 0,$ or -1.
- It is even. This eliminates $+3, +1, 0,$ and -1.

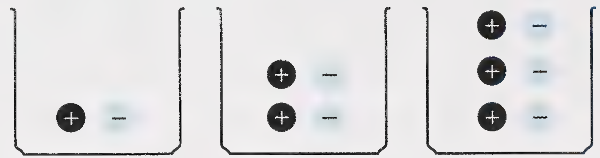
So, the number is $+2$.

Section 1: Activity 2

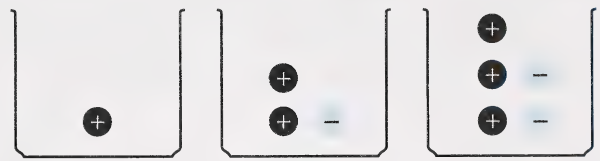
1. a. -3 b. $+4$ c. 0
   d. $+3$ e. -4 f. $+4$

2. Answers will vary. Three possible answers are given for each integer.

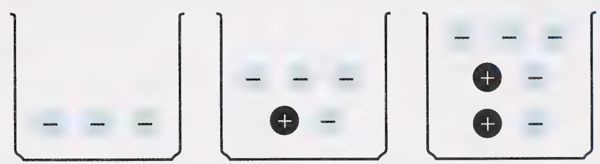
a.



b.



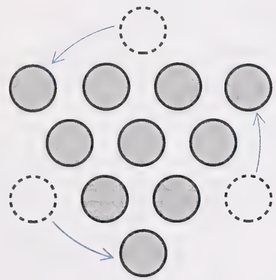
c.



3. You can use any number of zero pairs and not change the value.

Now Try This

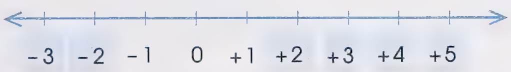
4. You can use coins or counters to solve this problem. Move the coins as shown in the following diagram.



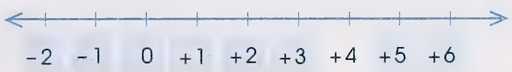
Section 1: Follow-up Activities

Extra Help

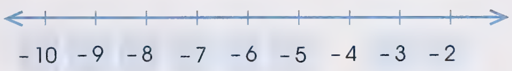
1. a.



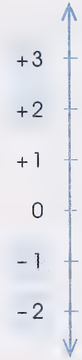
b.



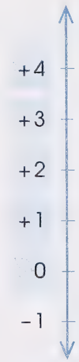
c.



d.



e.



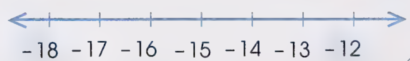
2. a. $6 < 16$ b. $-5 < -3$ c. $-2 > -20$

d. $-12 < 1$ e. $+4 < +5$ f. $0 > -3$

3. a. −4, −3, 0, 8, 12
   b. −10, −8, −5, −4, 0, +4, +6, +11
   c. −15, −6, −3, −1, +8, +11, +13

4. The scores of the golfers, from best score to worst, are −4, −2, −1, 0, +3, +5.

5. No, Cyril's conclusion is not true; $-18 < -12$.

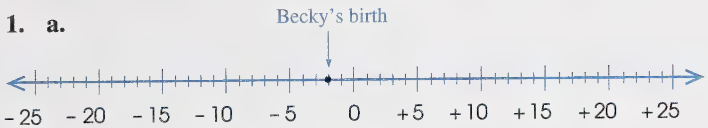


Numbers increase as you move from left to right on the number line.

6. Did you enjoy playing “Integer War”?

Enrichment

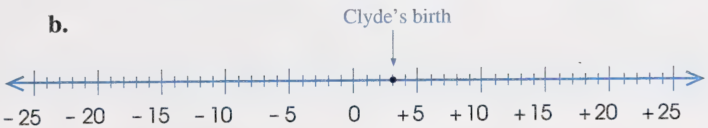
1. a.



1978 is 2 years before Justin’s birth.

The integer -2 represents the year of Becky’s birth.

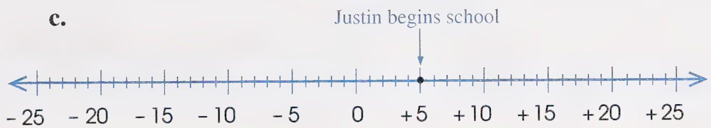
b.



1983 is 3 years after Justin’s birth.

The integer $+3$ represents the year of Clyde’s birth.

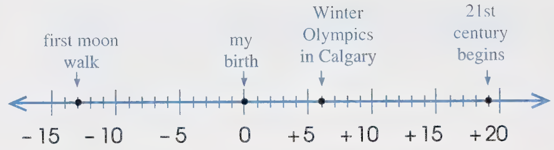
c.



1985 is 5 years after Justin’s birth.

The integer $+5$ represents the date of Justin starting school.

2. Answers will vary. This time line is for someone born in 1982.



Note: The Winter Olympics were held in 1988 (6 years after this person’s birth). Neil Armstrong and Buzz Aldrin walked on the Moon in 1969 (13 years before this person’s birth). The first day of the twenty-first century is on January 1, 2001 (19 years after this person’s birth).

3. The coin expert is correct. The coins dated 250 B.C. and -50 are forgeries. The practice of using the birth of Christ as a starting point for recording dates did not occur until after Christ’s death. So, a coin would not be dated with B.C. or with a negative sign.

Now Try This

4. **Step 1:** Guess 4.

$$\begin{aligned}&5\times4+2\times4\\&=20+8\\&=28\end{aligned}$$

$28<84$

This guess is too small.

Step 2: Because $84 \div 28 = 3$, the guess of 4 was 3 times too small. The number ought to be 3×4, or 12.

$$5 \times 12 + 2 \times 12$$
$$= 60 + 24$$
$$= 84$$

So, the number is 12.

Section 2: Activity 1

1. **a.** $(-400)+(+100)$ **b.** $(-3)+(-4)$
   c. $(+420)+(-100)$

2. **a.** The absolute value of the sum of two integers with like signs is equal to the total absolute value of the integers.

   b. The sign of the sum of two integers with like signs is the same as that of the integers being added.

3. **a.** $+7$ **b.** -11 **c.** -9 **d.** $+13$

4. $(+30)+(+45)=+75$

   Franz increased the speed of his car by 75 km/h.

5. $(-50)+(-40)=-90$

   Jasmine's account decreased $90.

6. **a.** The absolute value of the sum of two integers with unlike signs is equal to the difference of the absolute values of the integers.

   b. The sign of the sum of two integers with unlike signs is the same as the sign of the integer with the greater absolute value.

7. **a.** $+3$ **b.** -3 **c.** $+2$
   d. 0 **e.** -5 **f.** $+4$

8. $(+45)+(-10)=+35$

   The car moved forward 35 m.

9. $(+3)+(-8)=-5$

   The temperature fell 5°C.

10. **a.** $(-400)+(+100)=-300$

    The airplane descended 300 m altogether.

    b. $(-3)+(-4)=-7$

    The temperature fell 7°C altogether.

    c. $(+420)+(-100)=+320$

    Frank's account rose $320 altogether.

Now Try This

11. You can use logic and elimination to solve this problem.

Step 1: Ask yourself, what is the first digit?

- The digit 1 is before 3, but after 4. So, 1 and 3 are not the first digit.
- The digit 2 is after 4. So, 2 is not the first digit.
- The digit 5 is after 2. So, 5 is not the first digit.

By elimination, the first digit is 4.

	1	2	3	4	5
first	✗	✗	✗	✓	✗
second				✗	
third				✗	
fourth				✗	
fifth				✗	

Step 2: Ask yourself, what is the second digit?

- The digit 4 has been eliminated.
- The digit 1 is after 4, but the digit 2 is before 1. So, 1 is not the second digit.
- The digit 5 is after 2, but before 3. So, 5 and 3 are not the second digit.

By elimination, the second digit is 2.

	1	2	3	4	5
first	✗	✗	✗	✓	✗
second	✗	✓	✗	✗	✗
third		✗		✗	
fourth		✗		✗	
fifth		✗		✗	

Step 3: Ask yourself, what is the third digit?

- The digits 2 and 4 have been eliminated.
- The problem states that 5 is not the third digit.
- The digit 5 is before 3. So, 3 is not the third digit.

By elimination, 1 is the third digit.

	1	2	3	4	5
first	✗	✗	✗	✓	✗
second	✗	✓	✗	✗	✗
third	✓	✗	✗	✗	✗
fourth	✗	✗		✗	
fifth	✗	✗		✗	

Step 4: Ask yourself, what are the fourth and fifth digits?

- The digits 1, 2, and 4 have been eliminated.
- The digit 5 is before 3.

By elimination, 5 is the fourth digit and 3 is the fifth digit.

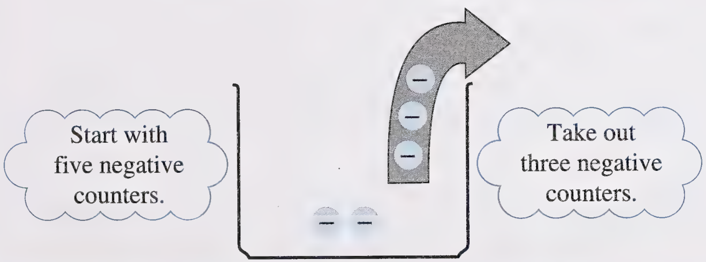
Therefore, the number is 42 153.

Note: You can also use the guess, check, and revise strategy to solve the problem. If you use this strategy, it might be helpful to use squares with the numbers on them. The squares can then be moved easily.

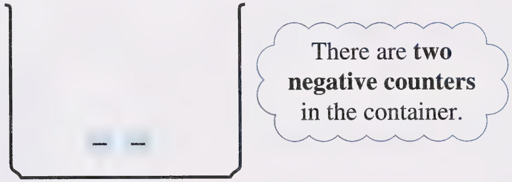
Section 2: Activity 2

1. a. $(+2)-(+5)$ **b.** $(+3)-(-2)$ **c.** $(-4)-(-3)$

2. a. Step 1: Model the expression. Start with five negative counters in the container; then take out three negative counters.

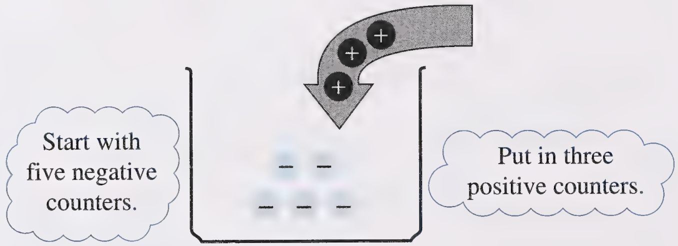


Step 2: Find the difference. **Hint:** Are the counters in the container negative or positive? How many are there?



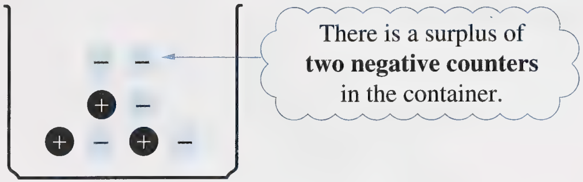
$\therefore\ (-5)-(-3)=-2$

b. Step 1: Model the expression. Start with five negative counters in the container; then add three positive counters.



Step 2: Because there is a combination of positive and negative counters, rearrange the counters making as many zero pairs as possible.

Step 3: Find the sum. **Hint:** Are the surplus counters in the container positive or negative? How many are there?



$\therefore\ (-5)+(+3)=-2$

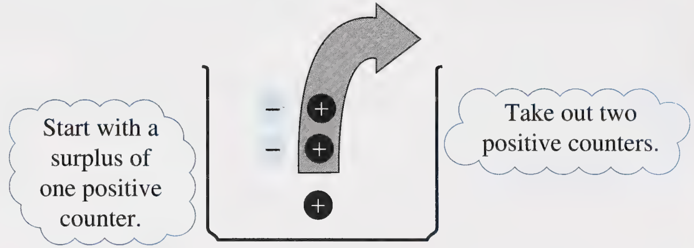
c. The first number in each expression is the same.

d. The operation signs in the expressions are the opposite of each other; the second number in the expression $(-5)+(+3)$ is the opposite of the second number in the expression $(-5)-(-3)$.

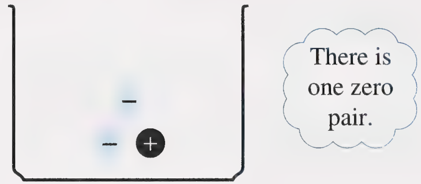
e. The value of the expression $(-5)-(-3)$ equals the value of the expression $(-5)+(+3)$.

3. a. Step 1: Model the expression. Start with one positive counter in the container; then take out two positive counters.

Hint: You will need sufficient zero pairs to do this.



Step 2: Because there is a combination of positive and negative counters, rearrange the counters making as many zero pairs as possible.



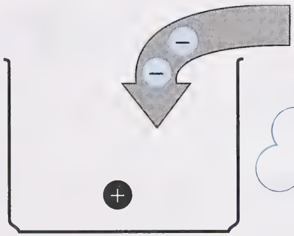
Step 3: Find the difference. **Hint:** Are the surplus counters in the container positive or negative? How many are there?

There is a surplus of **one negative counter** in the container.

$\therefore\ (+1)-(+2)=-1$

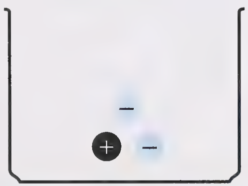
b. Step 1: Model the expression. Start with one positive counter; then add two negative counters.

Start with one positive counter.



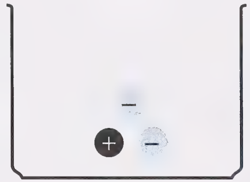
Put in two negative counters.

Step 2: Because there is a combination of positive and negative counters, rearrange the counters making as many zero pairs as possible.



There is one zero pair.

Step 3: Find the sum. **Hint:** Are the surplus counters in the container positive or negative? How many are there?



There is a surplus of **one negative counter** in the container.

$\therefore\ (+1)+(-2)=-1$

c. The first number in each expression is the same.

d. The operation signs in the expressions are the opposite of each other; the second number in the expression $(+1)-(+2)$ is the opposite of the second number in the expression $(+1)+(-2)$.

e. The value of the expression $(+1)-(+2)$ equals the value of the expression $(+1)+(-2)$.

4. a. $+2$ **b.** -2 **c.** -8

d. $+9$ **e.** -6 **f.** $+2$

5. a. $$\begin{aligned}(+2)-(+5)&=(+2)+(-5)\\&=-3\end{aligned}$$

The share price fell \$3.

b. $(+3)-(-2)=(+3)+(+2)$
$=+5$

The elevator rose 5 floors.

c. $(-4)-(-3)=(-4)+(+3)$
$=-1$

The submarine descended 1 km.

Now Try This

6. You can use the guess, check, and revise strategy to solve this problem. The integers are -6 and $+1$.

First Integer	3	2	1
Second Integer	−4	−5	−6

correct guess

7. You can use logic to solve this problem. The ship is floating on the water. As the tide rises, the ship rises. The ladder will also rise.

So, 5 m of the ladder will be submerged.

Section 2: Activity 3

1. a. $(+2)\times(+3)$ b. $(-2)\times(-3)$ c. $(-2)\times(+3)$

2. The sign of the product of two integers with like signs is always positive.

3. The sign of the product of two integers with unlike signs is always negative.

4. a. **Step 1:** Model $+5$.

+ + + + +

Step 2: To model the expression $(-1)\times(+5)$, replace the five positive counters with their opposites.

− − − − −

There are now **five negative counters**.

$\therefore\ (-1)\times(+5)=-5$

b. **Step 1:** Model -6.

− − − − − −

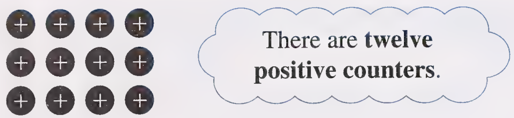
Step 2: To model the expression $(-1)\times(-6)$, replace the six negative counters with their opposites.

+ + + + + +

There are now **six positive counters**.

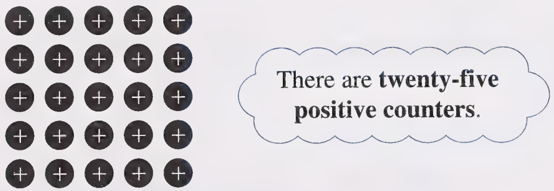
$\therefore\ (-1)\times(-6)=+6$

5. a.



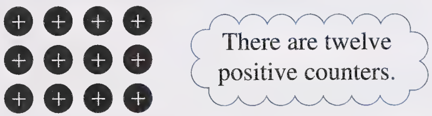
$\therefore\ (+3)\times(+4)=+12$

b.

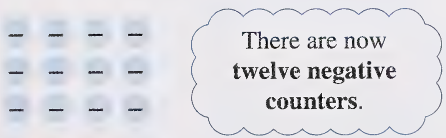


$\therefore\ (+5)\times(+5)=+25$

6. a. **Step 1:** Model the expression $(+3)\times(+4)$.

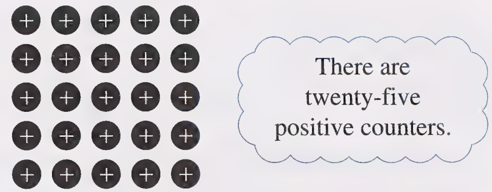


Step 2: There is **one** negative factor in the expression $(-3)\times(+4)$. So, replace the counters with their opposites **once**.

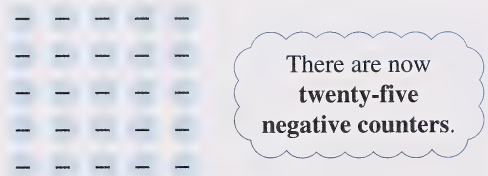


$\therefore\ (-3)\times(+4)=-12$

b. **Step 1:** Model the expression $(+5)\times(+5)$.

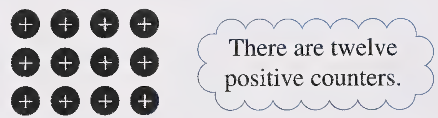


Step 2: There is **one** negative factor in the expression $(+5)\times(-5)$. So, replace the counters with their opposites **once**.

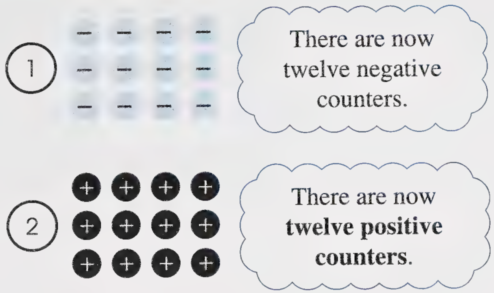


$\therefore\ (+5)\times(-5)=-25$

7. a. **Step 1:** Model the expression $(+3)\times(+4)$.

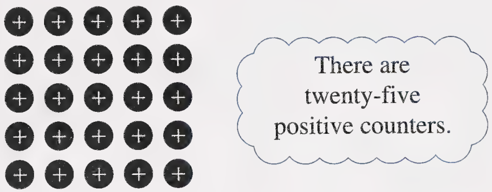


Step 2: There are **two** negative factors in the expression $(-3)\times(-4)$. So, replace the counters with their opposites **twice**.

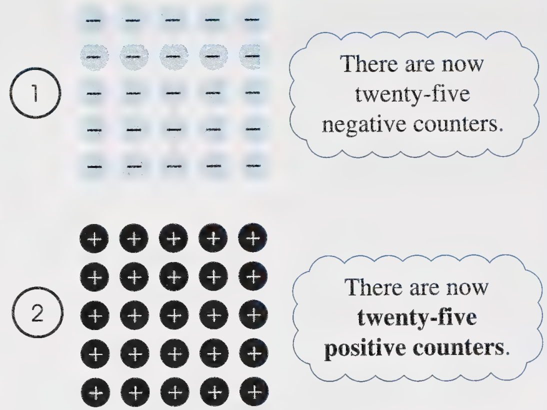


$\therefore\ (-3)\times(-4)=+12$

b. **Step 1:** Model the expression $(+5)\times(+5)$.



Step 2: There are **two** negative factors in the expression $(-5)\times(-5)$. So, replace the counters with their opposites **twice**.



$\therefore\ (-5)\times(-5)=+25$

8. **a.** The sign of the product is negative because the integers have unlike signs.

$(-4)\times(+3)=-12$

b. The sign of the product is positive because the integers have like signs.

$(-4)\times(-4)=+16$

c. The sign of the product is negative because the integers have unlike signs.

$(+5)\times(-2)=-10$

d. The sign of the product is negative because the integers have unlike signs.

$$\begin{array}{r} (+2) \\ \times\ (-6) \\ \hline -12 \end{array}$$

e. The sign of the product is positive because the integers have like signs.

$$\begin{array}{r} (+3) \\ \times\ (+9) \\ \hline +27 \end{array}$$

f. The sign of the product is positive because the integers have like signs.

$$\begin{array}{r} (-7) \\ \times\ (-3) \\ \hline +21 \end{array}$$

9. a. $(+2)\times(+3)=+6$

Michelle will be 6 m above the water line.

b. $(-2)\times(-3)=+6$

Michelle was 6 m above the water line.

c. $(-2)\times(+3)=-6$

Michelle will be 6 m below the water line.

Now Try This

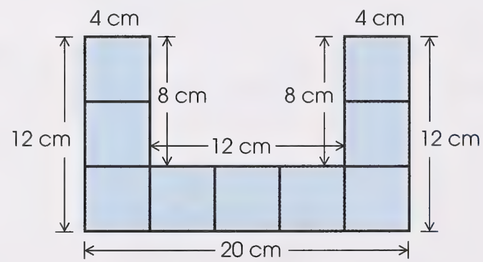
10. You can use logical reasoning to solve this problem.

If there are 9 squares and the total area is $144\ \text{cm}^2$, then the area of one square is $16\ \text{cm}^2$.

$144 \div 9 = 16$

If the area of one square is $16\ \text{cm}^2$, then the length of one side of each square is 4 cm.

$16 = 4 \times 4$



$4+12+20+12+4+8+12+8=80$

The perimeter is 80 cm.

11. You can find and apply a pattern to help you solve this problem.

Pattern	A	B	C	D	Pattern
+7	1	2	3	4	+1
+1	8	7	6	5	+2
+7	9	10	11	12	+7
	16	15	14	13	

The numbers in Column A will be 1, 8, 9, 16, 17, 24, 25, 32, 33, 40, 41, 48, 49, 56, 57, 64, 65, 72, 73, 80, 81, 88, 89, 96, 97, 104,….

The numbers in Column D will be 4, 5, 12, 13, 20, 21, 28, 29, 36, 37, 44, 45, 52, 53, 60, 61, 68, 69, 76, 77, 84, 85, 92, 93, 100,….

So, the number 100 will appear in Column D.

Section 2: Activity 4

1. a. $(+10) \div (+5)$ b. $(-30) \div (+6)$
   c. $(-2000) \div (-100)$

2. The sign of the quotient of two integers with like signs is positive.

3. The sign of the quotient of two integers with unlike signs is negative.

4. a. The sign of the quotient is positive because the integers have like signs.

   $$(-8) \div (-2) = +4$$

   b. The sign of the quotient is negative because the integers have unlike signs.

   $$(-18) \div (+2) = -9$$

   c. The sign of the quotient is positive because the integers have like signs.

   $$(-3)\overline{)(-15)}^{\,+5}$$

   d. The sign of the quotient is negative because the integers have unlike signs.

   $$(+5)\overline{)(-10)}^{\,-2}$$

5. a. $(+10) \div (+5) = +2$

   The temperature rose 2°C per minute.

   b. $(-30) \div (+6) = -5$

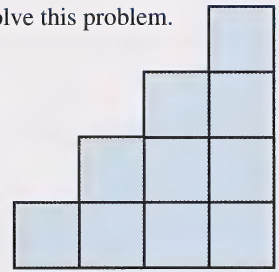
   The temperature fell 5°C per hour.

   c. $(-2000) \div (-100) = +20$

   It will take 20 minutes.

Now Try This

6. You can use logic and a diagram to solve this problem. Divide the figure into squares.



Step 1: Find the area of each square. Each square is 3 units by 3 units.

$3 \times 3 = 9$

Each square has an area of 9 square units.

Step 2: Find the total area of the figure. The figure contains 10 squares.

$10 \times 9 = 90$

So, the area of the figure is 90 square units.

Section 2: Activity 5

1. a. $(-2)\times(-6) \div(-3)$ ← Multiply.
$= (+12)\div(-3)$ ← Divide.
$= -4$

b. $(+4)+ (-2)\times(-5)$ ← Multiply.
$= (+4)+(+10)$ ← Add.
$= +14$

c. $(-3) -(-4) +(-2)$ ← Change subtraction to addition.
$= (-3)+(+4) +(-2)$ ← Add.
$= (+1)+(-2)$ ← Add.
$= -1$

d. $(+14)\div(-2) + (+6)\times(-3)$ ← Divide; then multiply.
$= (-7)+(-18)$ ← Add.
$= -25$

e. $(-2)\times(+3) -(-2)$ ← Change subtraction to addition.
$= (-2)\times(+3) +(+2)$ ← Multiply.
$= (-6)+(+2)$ ← Add.
$= -4$

f. $(+7) -(+8) \times(-3)$ ← Change subtraction to addition.
$= (+7)+ (-8)\times(-3)$ ← Multiply.
$= (+7)+(+24)$ ← Add.
$= +31$

g. $(-4) - (+4) \div (-2)$ ← Change subtraction to addition.

$= (-4) + (-4) \div (-2)$ ← Divide.

$= (-4) + (+2)$ ← Add.

$= -2$

h. $(+9) \div (-3) + (+5) \times (-2) - (+1)$ ← Change subtraction to addition.

$= (+9) \div (-3) + (+5) \times (-2) + (-1)$ ← Divide; then multiply.

$= (-3) + (-10) + (-1)$ ← Add.

$= (-13) + (-1)$ ← Add.

$= -14$

2. a. $(-6) \div [(+2) - (-1)]$ ← Work in square brackets first; change subtraction to addition.

$= (-6) \div [(+2) + (+1)]$ ← Add.

$= (-6) \div (+3)$ ← Divide.

$= -2$

b. $(-2) - [(+6) + (-3)]$ ← Work in square brackets first; add.

$= (-2) - (+3)$ ← Change subtraction to addition.

$= (-2) + (-3)$ ← Add.

$= -5$

3.
a. $5 + 6$ **b.** $-3 + 4$ **c.** $-3 + (-2)$
d. $-2 - 5$ **e.** $3 - 10$ **f.** $-5 - (-10)$
g. -8×12 **h.** $3 \times (-2)$ **i.** -2×5
j. $6 \div (-2)$ **k.** $30 \div 5$ **l.** $-25 \div (-5)$
m. $-6 \times [2 - (-1)]$ **n.** $-2 - [6 + (-3)]$

4.
a. $(-6) + (+9)$ **b.** $(-4) - (-6)$
c. $(-4) \times (+3)$ **d.** $(+8) \div (-4)$
e. $(-30) \div (+6)$ **f.** $(+2) - (+9)$
g. $(-5) \div [(+2) - (+3)]$ **h.** $(+8) - [(+4) - (+6)]$
i. $(+5) \times [(-3) + (+2)]$

Now Try This

5. You can use patterns and logic to solve this problem.

> This type of clock reads 9:00 twice a day—in the morning (9 A.M.) and in the evening (9 P.M.).

Step 1: Use ratio and proportion to find the clock gain if the clock gains 5 minutes every hour.

$$\frac{5}{1} \overset{\times 12}{=} \frac{60}{12}$$

The clock gains 60 minutes (1 hour) every 12 hours.

Step 2: Use patterns to find when the clock will give the correct time again.

Pattern (in hours)

+12
+12
+12
+12
+12
+12
+12
+12
+12
+12
+12
+12
+144

Time on Clock	Correct Time
9:00	9:00
10:00	9:00
11:00	9:00
12:00	9:00
1:00	9:00
2:00	9:00
3:00	9:00
4:00	9:00
5:00	9:00
6:00	9:00
7:00	9:00
8:00	9:00
9:00	9:00

The clock will give the correct time in 144 h or in 6 days.

Section 2: Activity 6

1. a. 2 7 +/− + 8 6 =

59.

b. 1 3 2 +/− − 5 6 +/− =

-76.

c. 2 5 +/− × 1 5 +/− =

375.

d. 2 4 5 +/− ÷ 3 5 =

-7.

2. a. (1 5 0 +/− + 6) ÷ 1 2 =

-12.

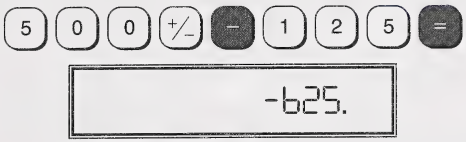
b. 1 3 − (2 +/− + 1 0 +/−) =

25.

3. **Step 1:** Describe the situation with a mathematical expression.

$$(-500)-(+125)$$

Step 2: Evaluate the expression using a calculator.

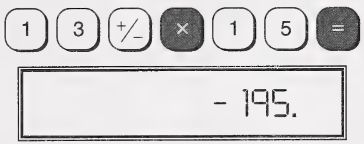


The miners descended 625 m.

4. **Step 1:** Describe the situation with a mathematical expression.

$$(-13)\times(+15)$$

Step 2: Evaluate the expression using a calculator.

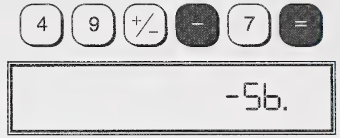


The volume of water dropped 195 L.

5. **Step 1:** Describe the situation with a mathematical expression.

$$(-49)-(+7)$$

Step 2: Evaluate the expression using a calculator.



The temperature dropped 56°C; the change was 56 degrees.

Now Try This

6. You can use logic and the diagrams to solve this problem.

Step 1: Count the number of segments in the perimeter of Figure A.

There are 16 segments.

Step 2: Calculate the length of each segment.

$$48 \div 16 = 3$$

Each segment is 3 units long.

Step 3: Count the number of segments in the perimeter of Figure B.

There are 20 segments.

Step 4: Calculate the perimeter of Figure B.

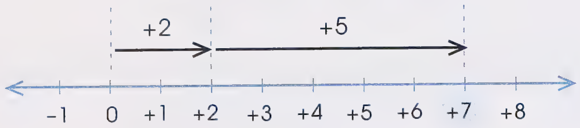
$$3 \times 20 = 60$$

The perimeter of Figure B is 60 units.

Section 2: Follow-up Activities

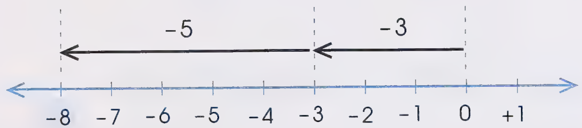
Extra Help

1. a.



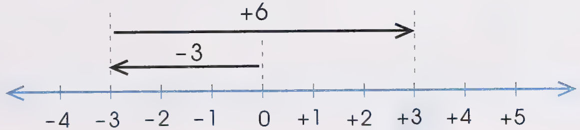
$\therefore\ (+2)+(+5)=+7$

b.



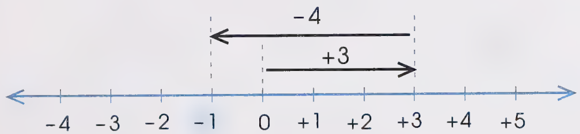
$\therefore\ (-3)+(-5)=-8$

c.



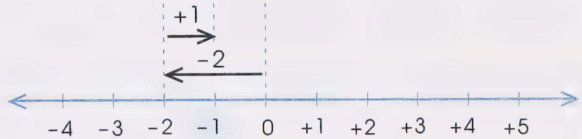
$\therefore\ (-3)+(+6)=+3$

d.



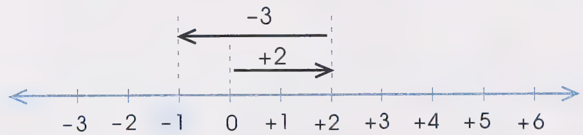
$\therefore\ (+3)+(-4)=-1$

e.



$\therefore\ (-2)+(+1)=-1$

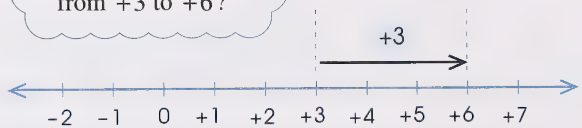
f.



$\therefore\ (+2)+(-3)=-1$

2. a.

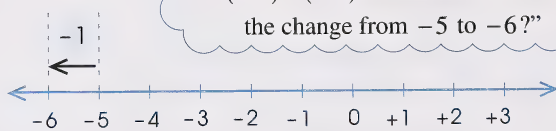
$(+6)-(+3)$ can mean, "What is the change from $+3$ to $+6$?"



$\therefore\ (+6)-(+3)=+3$

b.

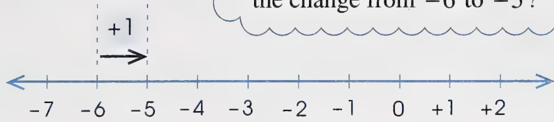
$(-6)-(-5)$ can mean, "What is the change from -5 to -6?"



$\therefore\ (-6)-(-5)=-1$

c.

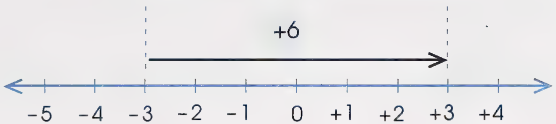
$(-5)-(-6)$ can mean, "What is the change from -6 to -5?"



$\therefore\ (-5)-(-6)=+1$

d.

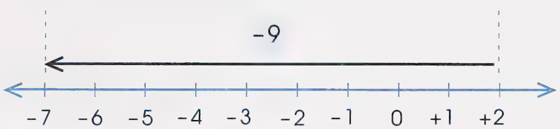
$(+3)-(-3)$ can mean, "What is the change from -3 to $+3$?"



$\therefore\ (+3)-(-3)=+6$

e.

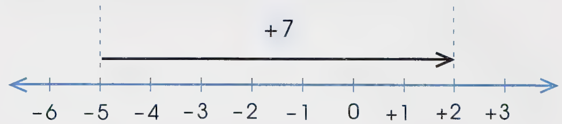
$(-7)-(+2)$ can mean, "What is the change from $+2$ to -7?"



$\therefore\ (-7)-(+2)=-9$

f.

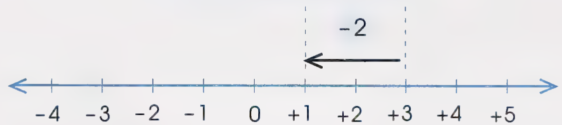
$(+2)-(-5)$ can mean, "What is the change from -5 to $+2$?"



$\therefore\ (+2)-(-5)=+7$

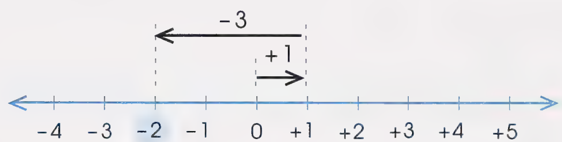
3. a. Step 1: Find the difference.

$(+1)-(+3)$ can mean, "What is the change from $+3$ to $+1$?"



$\therefore\ (+1)-(+3)=-2$

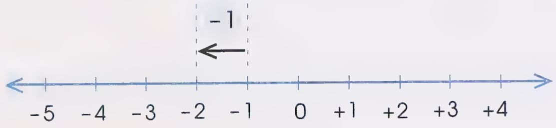
Step 2: Find the sum.



$\therefore\ (+1)+(-3)=-2$

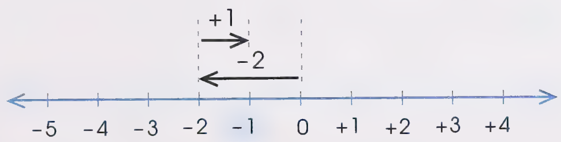
b. **Step 1:** Find the difference.

$(-2)-(-1)$ can mean, "What is the change from -1 to -2?"



$\therefore\ (-2)-(-1)=-1$

Step 2: Find the sum.

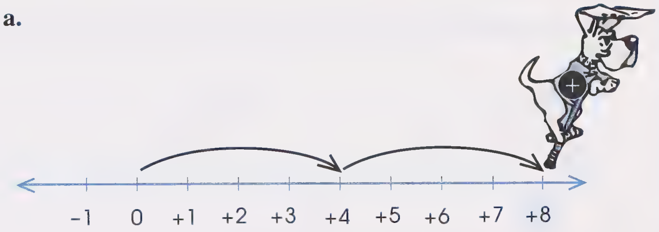


$\therefore\ (-2)+(+1)=-1$

4. **a.** They are the same.

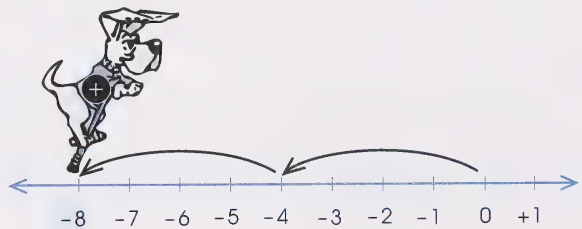
b. Instead of subtracting an integer, you can add the opposite integer.

5. **a.**



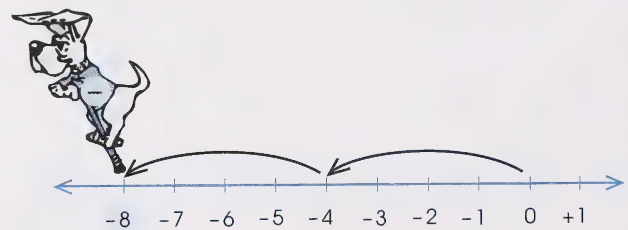
$\therefore\ (+2)\times(+4)=+8$

b.



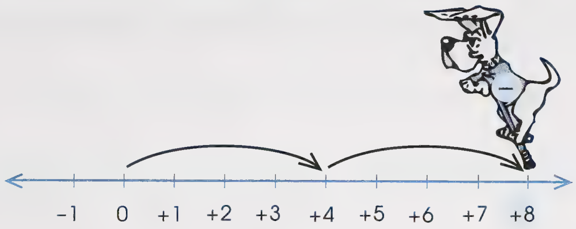
$\therefore\ (+2)\times(-4)=-8$

c.



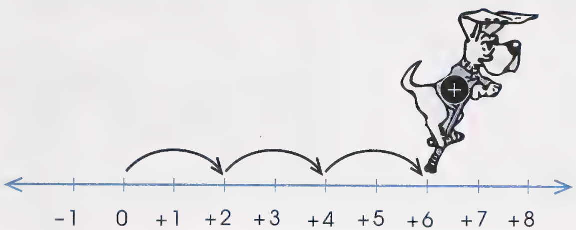
$\therefore\ (-2)\times(+4)=-8$

d.



$\therefore\ (-2)\times(-4)=+8$

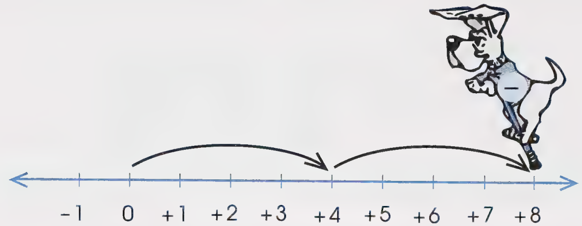
6. a. $(+6)\div(+3)=$ is the same as $(+3)\times$ $=+6$.



Droopy must jump **forward** in three steps of **2**.

$\therefore\ (+6)\div(+3)=+2.$

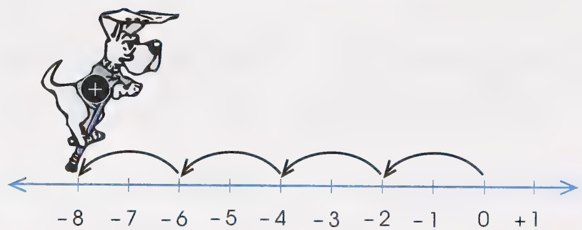
b. $(+8)\div(-2)=$ is the same as $(-2)\times$ $=+8$.



Droopy must jump **backwards** in two steps of **4**.

$\therefore\ (+8)\div(-2)=-4$

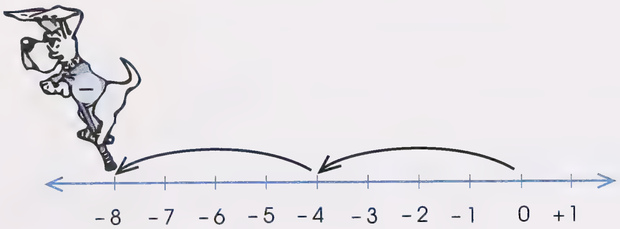
c. $(-8)\div(+4)=$ is the same as $(+4)\times$ $=-8$.



Droopy must jump **backwards** in four steps of **2**.

$\therefore\ (-8)\div(+4)=-2$

d. $(-8) \div (-2) = \quad$ is the same as $(-2) \times \quad = -8.$



Droopy must jump **forward** in two steps of **4**.

$\therefore\ (-8) \div (-2) = +4$

Enrichment

1. **a.** $-19°C$ **b.** $-25°C$ **c.** $-31°C$ **d.** $-37°C$

2. **a.** $(-19)-(-10)=(-19)+(+10)$
$=-9$

The wind-chill equivalent temperature is 9°C colder.

b. $(-25)-(-15)=(-25)+(+15)$
$=-10$

The wind-chill equivalent temperature is 10°C colder.

c. $(-31)-(-20)=(-31)+(+20)$
$=-11$

The wind-chill equivalent temperature is 11°C colder.

d. $(-37)-(-25)=(-37)+(+25)$
$=-12$

The wind-chill equivalent temperature is 12°C colder.

3. **a.** The corresponding wind-chill equivalent temperatures are $-17°C$ and $-37°C$.

b. **Step 1:** Find the decrease in the actual thermometer reading.

$$(-20)-(-5)=-20+(+5)$$
$$=-15$$

The actual thermometer reading decreased 15°C.

Step 2: Find the decrease in the wind-chill equivalent temperature.

$$(-37)-(-17)=(-37)+(+17)$$
$$=-20$$

The wind-chill equivalent temperature decreased 20°C.

Step 3: Compare the decreases.

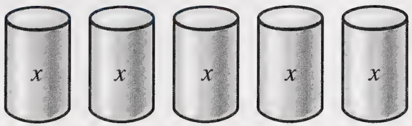
$$-20 < -15$$

The wind-chill equivalent temperatures decreased more.

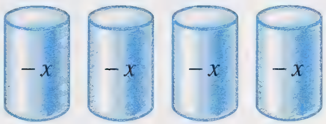
4. The wind-chill equivalent temperature is $-27\,°C$.

Section 3: Activity 1

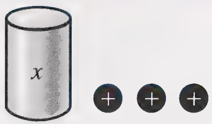
1. a.



b.

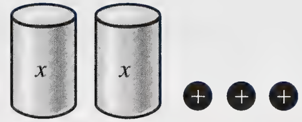


c.

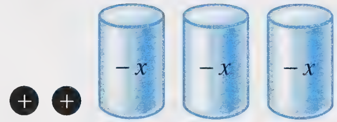


d.

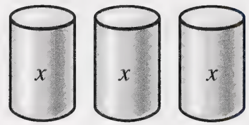
e.



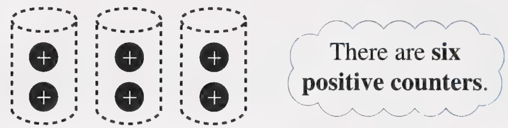
f.



2. $3x$ can be modelled like this.



a. If $x = +2$, then each cylinder labelled x can be replaced by two positive counters.



If $x = +2$, then $3x = +6$.

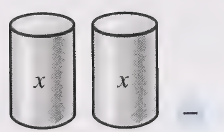
b. If $x = -1$, then each cylinder labelled x can be replaced by one negative counter.

If $x = -1$, then $3x = -3$.

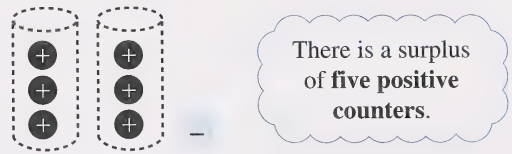
c. If $x = 0$, then each cylinder labelled x can be replaced by one zero pair.

If $x = 0$, then $3x = 0$.

3. $2x - 1$ can be modelled like this.

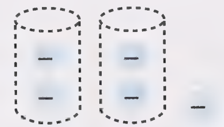


a. If $x = +3$, then each cylinder labelled x can be replaced by three positive counters.



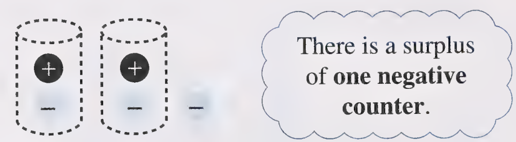
If $x = +3$, then $2x - 1 = 5$.

b. If $x = -2$, then each cylinder labelled x can be replaced by two negative counters.



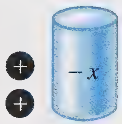
If $x = -2$, then $2x - 1 = -5$.

c. If $x = 0$, then each cylinder labelled x can be replaced by a zero pair.

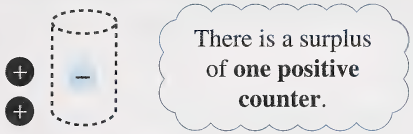


If $x = 0$, then $2x - 1 = -1$.

4. $2-x$ can be modelled like this.

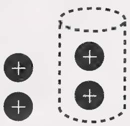


a. If $x=+1$, then $-x=-1$; so, the cylinder labelled $-x$ can be replaced by one negative counter.



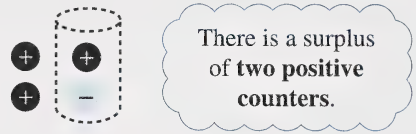
If $x=+1$, then $2-x=+1$.

b. If $x=-2$, then $-x=+2$; so, the cylinder labelled $-x$ can be replaced by two positive counters.



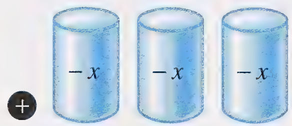
If $x=-2$, then $2-x=+4$.

c. If $x=0$, then $-x=0$; so, the cylinder labelled $-x$ can be replaced by a zero pair.

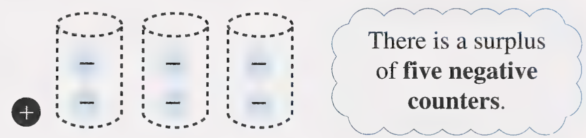


If $x=0$, then $2-x=+2$.

5. $1-3x$ can be labelled like this.

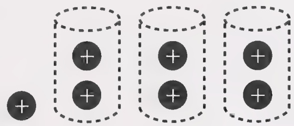


a. If $x=+2$, then $-x=-2$; so, each cylinder labelled $-x$ can be replaced by two negative counters.



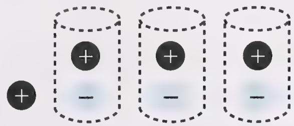
If $x=+2$, then $1-3x=-5$.

b. If $x=-2$, then $-x=+2$; so, each cylinder labelled $-x$ can be replaced by two positive counters.



If $x=-2$, then $1-3x=+7$.

c. If $x=0$, then $-x=0$; so, each cylinder labelled $-x$ can be replaced by a zero pair.



There is a surplus of **one positive counter**.

If $x=0$, then $1-3x=1$.

6. a. If $a=-3$, then

$$\begin{aligned}&4a-1\\&=4\times(-3)-1\\&=(-12)-1\\&=(-12)-(+1)\\&=(-12)+(-1)\\&=-13\end{aligned}$$

b. If $a=-2$, then

$$\begin{aligned}&4a-1\\&=4\times(-2)-1\\&=(-8)-1\\&=(-8)+(-1)\\&=-9\end{aligned}$$

c. If $a=-1$, then

$$\begin{aligned}&4a-1\\&=4\times(-1)-1\\&=(-4)-1\\&=(-4)+(-1)\\&=-5\end{aligned}$$

d. If $a=0$, then

$$\begin{aligned}&4a-1\\&=4\times0-1\\&=0-1\\&=0+(-1)\\&=-1\end{aligned}$$

7. a. If $b=2$, then

$$\begin{aligned}&2-5b\\&=2-5\times2\\&=2-10\\&=(+2)-(+10)\\&=(+2)+(-10)\\&=-8\end{aligned}$$

b. If $b=-3$, then

$$\begin{aligned}&2-5b\\&=2-5\times(-3)\\&=2-(-15)\\&=(+2)+(+15)\\&=17\end{aligned}$$

c. If $b=0$, then

$$\begin{aligned}&2-5b\\&=2-5\times0\\&=2-0\\&=2\end{aligned}$$

d. If $b=1$, then

$$\begin{aligned}&2-5b\\&=2-5\times1\\&=2-5\\&=(+2)-(+5)\\&=(+2)+(-5)\\&=-3\end{aligned}$$

Now Try This

8. You can use the diagram and logical reasoning to solve the problem.

- The area of Rectangle *B* is 25 m^2. So, the length must be 5 m and the width must be 5 m.
- The area of Rectangle *C* is 30 m^2. Rectangle *C* shares a side with Rectangle *B*. So Rectangle *C* must be 5 m by 6 m.
- The area of Rectangle *A* is 40 m^2. Rectangle *A* shares a side with Rectangle *B*. So, Rectangle *A* must be 5 m by 8 m.
- Rectangle *D* shares a side with Rectangle *C* and another side with Rectangle *A*. So, Rectangle *D* must be 6 m by 8 m.

$$\begin{aligned} A &= \ell w \\ &= 8 \times 6 \\ &= 48 \end{aligned}$$

So, the area of Rectangle *D* is 48 m^2.

Section 3: Activity 2

1. **a.** This was done to isolate the variable.
b. This was done to maintain the equality.

2. To solve equations of the form $x + a = b$, where x is a variable and a and b are integers, add the opposite of a to each side of the equation; then simplify.

3. **a.** -2 **b.** -9 **c.** $+5$ **d.** $+2$

4. **a.**

$$\begin{aligned} x \cancel{+2} &= +7 \\ \cancel{-2} &\;\; -2 \\ \hline x \quad &= +5 \end{aligned}$$

LS	RS
$x + 2$	7
$= 5 + 2$	
$= 7$	

LS = RS

b.

$$\begin{aligned} m + 9 &= -13 \\ m + (+9) &= -13 \\ m + \cancel{(+9)} + \cancel{(-9)} &= (-13) + (-9) \\ m &= -22 \end{aligned}$$

LS	RS
$m + 9$	-13
$= -22 + 9$	
$= (-22) + (+9)$	
$= -13$	

LS = RS

c.
$$\begin{aligned} t \cancel{-5} &= +7 \\ \cancel{+5} \quad &\;\; +5 \\ \hline t &= +12 \end{aligned}$$

LS	RS
$t-5$	7
$=12-5$	
$=12+(-5)$	
$=7$	

LS = RS

d.
$$\begin{aligned} y-2 &= -8 \\ y+(-2) &= -8 \\ y+\cancel{(-2)}+\cancel{(+2)} &= (-8)+(+2) \\ y &= -6 \end{aligned}$$

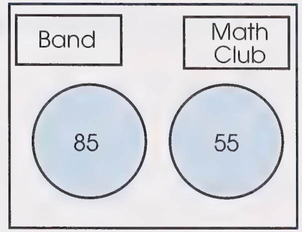
LS	RS
$y-2$	-8
$=(-6)-2$	
$=(-6)-(+2)$	
$=(-6)+(-2)$	
$=-8$	

LS = RS

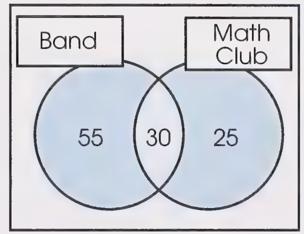
Now Try This

5. You can use logical reasoning and a Venn diagram to solve this problem.

Step 1: The band has 85 students. Use a circle to represent the band. The math club has 55 students. Use a second circle to represent the math club.

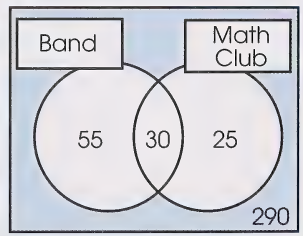


Step 2: There are 30 students who belong to both the band and the math club. Calculate the number of students who belong only to the band and only to the math club.



$$85-30=55$$
$$55-30=25$$

Step 3: There are 400 students at Parkview Junior High. Calculate the number of students who belong to neither the band nor the math club.



$$\begin{aligned} &400-(55+30+25) \\ &=400-110 \\ &=290 \end{aligned}$$

So, 290 students belong to neither the band nor the math club.

Section 3: Follow-up Activities

Extra Help

1. **Flow Chart**

x → Add 5. → 8

Reverse Flow Chart

8 → Subtract 5. → 3

$8 - 5 = 3$

$\therefore\ x = 3$

2. **Flow Chart**

n → Subtract 2. → 10

Reverse Flow Chart

10 → Add 2. → 12

$10 + 2 = 12$

$\therefore\ n = 12$

3. **Flow Chart**

y → Add 3. → 1

Reverse Flow Chart

1 → Subtract 3. → -2

$1 - 3 = (+1) + (-3)$
$= -2$

$\therefore\ y = -2$

4. **Flow Chart**

r → Subtract 5. → -2

Reverse Flow Chart

-2 → Add 5. → 3

$(-2) + (+5) = 3$

Enrichment

The answer to the puzzle is **QUESTIONABLE** (question a bull).

"Integer War" Cards: Set 1

+1	+2	+3	+4	+5
+6	+7	+8	+9	+10
+11	+12	+13	ABSOLUTE VALUE	ABSOLUTE VALUE

“Integer War” Cards: Set 1—*Continued*

−1	−2	−3	−4	−5
−6	−7	−8	−9	−10
−11	−12	−13	ABSOLUTE VALUE	ABSOLUTE VALUE

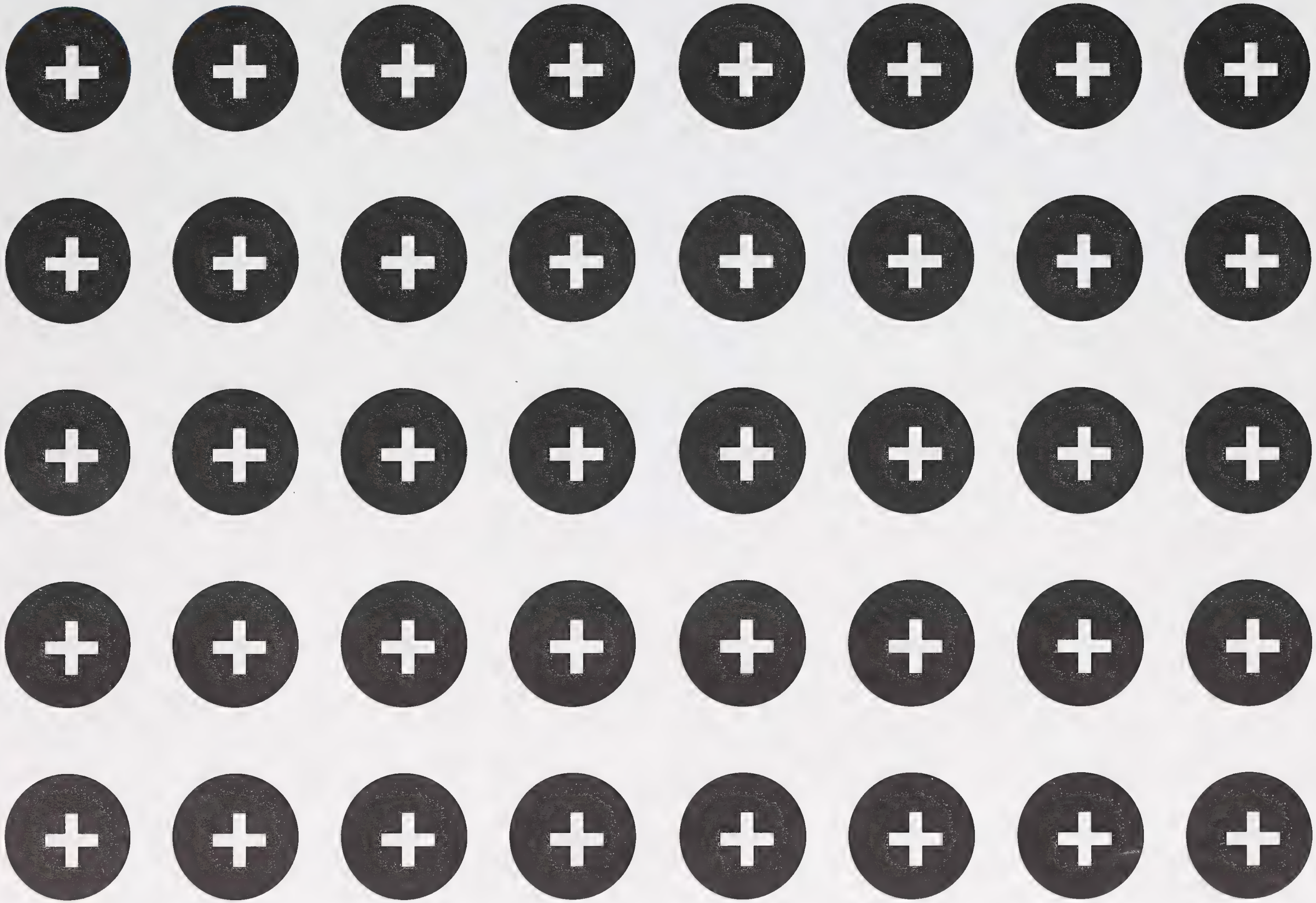
"Integer War" Cards: Set 2

+1	+2	+3	+4	+5
+6	+7	+8	+9	+10
+11	+12	+13	ABSOLUTE VALUE	ABSOLUTE VALUE

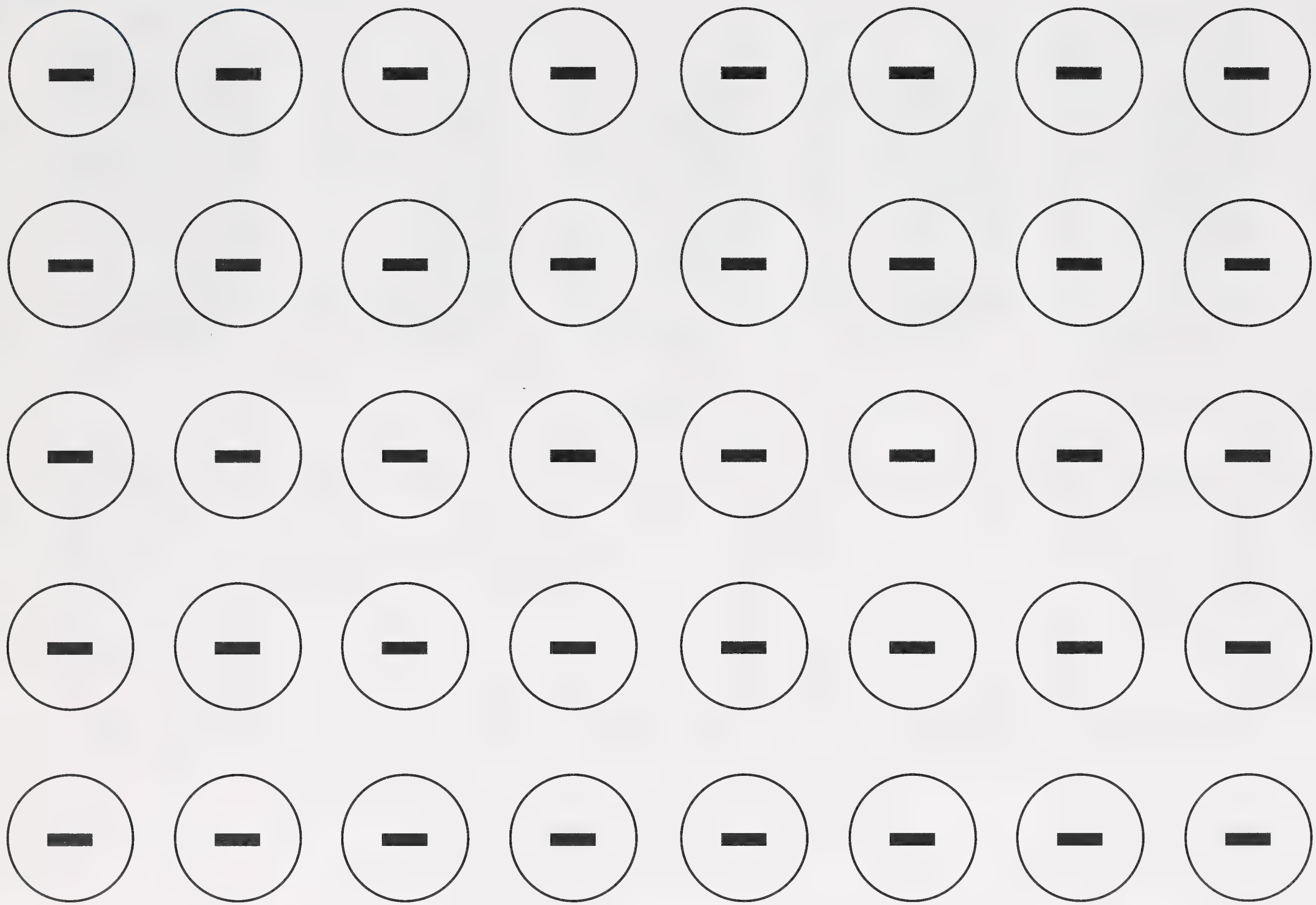
"Integer War" Cards: Set 2—*Continued*

−1	−2	−3	−4	−5
−6	−7	−8	−9	−10
−11	−12	−13	ABSOLUTE VALUE	ABSOLUTE VALUE

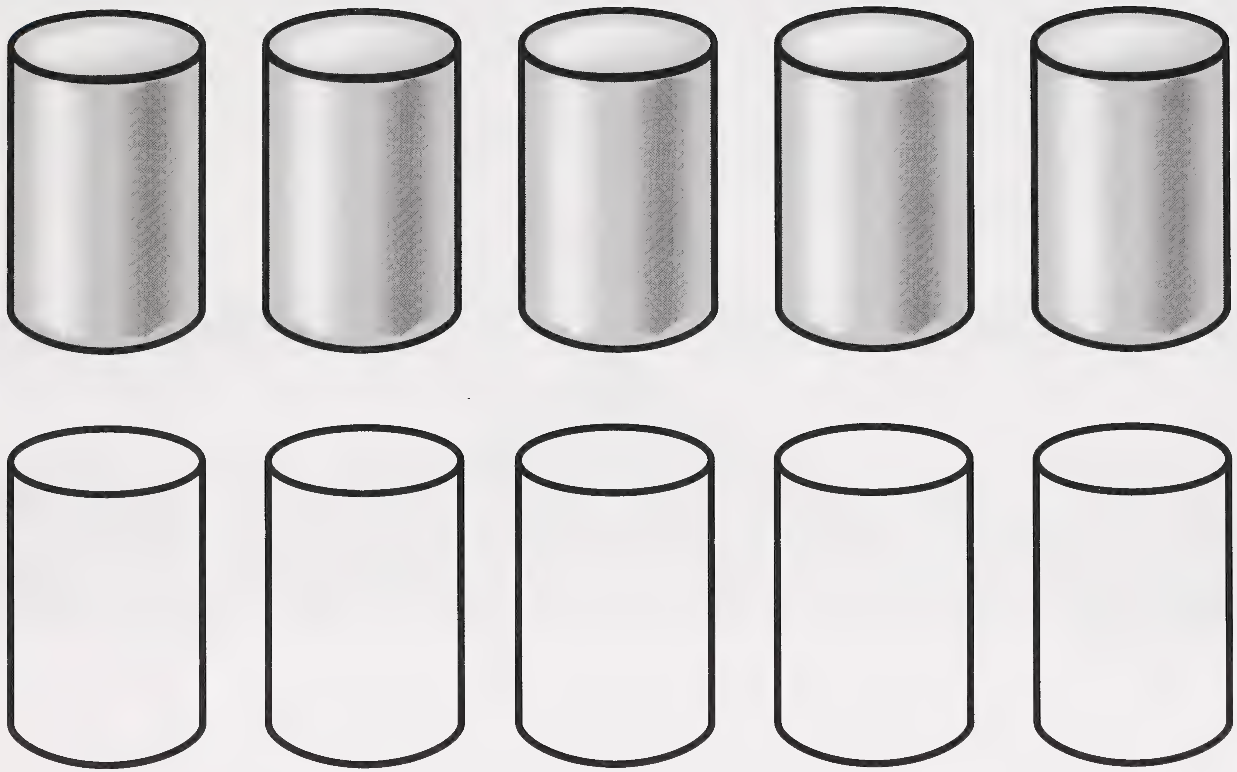
Positive Counters



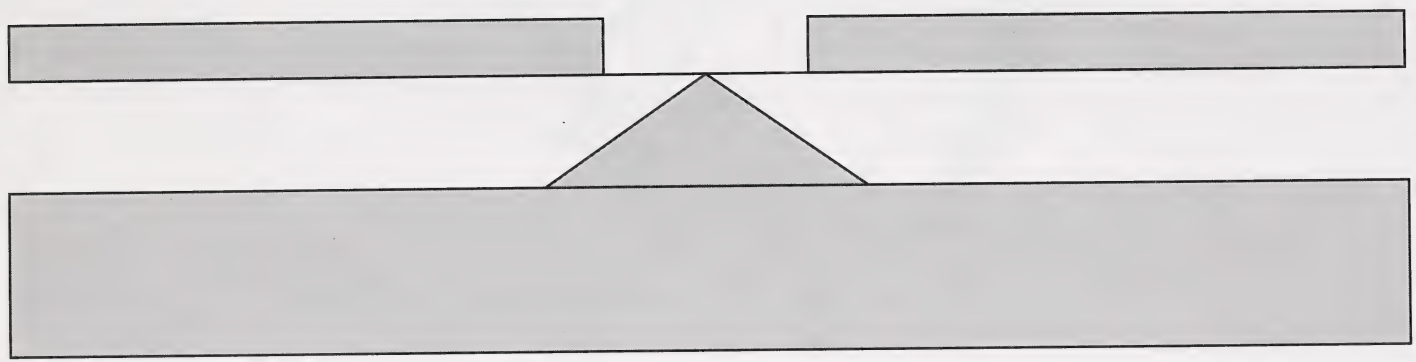
Negative Counters



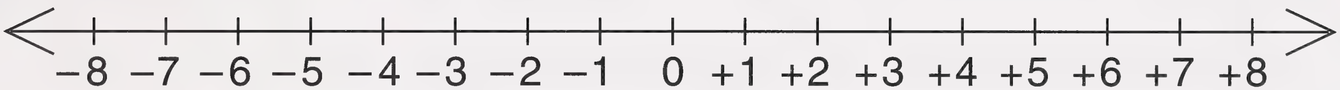
Cylinders



Equation Scale



Droopy Cut-out Learning Aids



What Do You Call It When Police Interrogate a Cow's Husband?[1]

Solve each problem and find the solution in the rectangle below. Cross out the box containing that solution. When you finish, there will be six boxes not crossed out. Print the letters from these boxes in the spaces at the bottom of the page.

1. Eight more than a number is 20. Find the number.
2. Twelve less than a number is -3. Find the number.
3. Three more than a number is -5. Find the number.
4. Nine less than a number is -24. Find the number.
5. If 10 is subtracted from a number, the result is 23. Find the number.
6. If 32 is added to a number, the result is -4. Find the number.
7. If a number is increased by 6, the result is 50. Find the number.
8. If a number is decreased by 16, the result is -2. Find the number.
9. The length of a rectangular lot is 78 m. This is 51 m more than the width. What is the width?
10. Andy hit 14 home runs this season. If this is 9 fewer than he hit last season, how many home runs did he hit last season?
11. Jennifer added \$120 to her savings account during July. If this brought her balance to \$700, how much had she saved previously?
12. The temperature in Frostburg is -7°C. This is 18°C less than the temperature in Coldspot. Find the temperature in Coldspot.
13. After 9 new members joined the ski club, there were 38 members. How many members had been in the club previously?
14. The altitude of a submarine is -60 m. If this is 25 m less than its previous altitude, what was its previous altitude?

CO	IN	JA	WS	LK	QU	IT	SH	AM	ES
14	33	−35 m	\$580	12	−75 m	29	−15	9	\$565
OO	TI	ME	ON	ST	TO	AB	OP	ED	LE
−36	8°C	27 m	31	−8	11°C	17	44	23	32 m

[1] 1978 Creative Publications for puzzle from *Pre-Algebra with Pizzazz.*

Mathematics 7
Student Module Booklet
Module 5
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